Lesson 4.3 Exercises, pages 298–304

A

4. Use these tables to determine each value below.

x	$f(x)$
−2	4
−1	2
0	0
1	−2
2	−4

x	$g(x)$
−4	−3
−2	1
0	5
2	9
4	13

a) $f(g(-2))$

From the 2nd table: $g(-2) = 1$
From the 1st table: $f(1) = -2$
So, $f(g(-2)) = -2$

b) $g(f(-2))$

From the 1st table: $f(-2) = 4$
From the 2nd table: $g(4) = 13$
So, $g(f(-2)) = 13$

c) $f(f(-1))$

From the 1st table: $f(-1) = 2$
From the 1st table: $f(2) = -4$
So, $f(f(-1)) = -4$

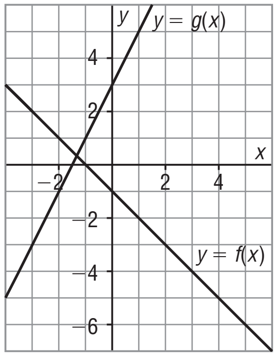
d) $g(f(0))$

From the 1st table: $f(0) = 0$
From the 2nd table: $g(0) = 5$
So, $g(f(0)) = 5$

5. Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below.

a) $f(g(-1))$

From the graph of $y = g(x)$, $g(-1) = 1$
From the graph of $y = f(x)$, $f(1) = -2$
So, $f(g(-1)) = -2$



b) $g(f(-2))$

From the graph of $y = f(x)$, $f(-2) = 1$
From the graph of $y = g(x)$, $g(1) = 5$
So, $g(f(-2)) = 5$

c) $g(g(-2))$

From the graph of $y = g(x)$, $g(-2) = -1$
From the graph of $y = g(x)$, $g(-1) = 1$
So, $g(g(-2)) = 1$

d) $f(g(1))$

From the graph of $y = g(x)$, $g(1) = 5$
From the graph of $y = f(x)$, $f(5) = -6$
So, $f(g(1)) = -6$

6. Given the functions $f(x) = 3x + 1$ and $g(x) = x^2 - 4$, determine each value.

a) $f(g(2))$

$g(2) = 2^2 - 4$
$= 0$
$f(g(2)) = f(0)$
$= 3(0) + 1$
$= 1$
So, $f(g(2)) = 1$

b) $g(f(2))$

$f(2) = 3(2) + 1$
$= 7$
$g(f(2)) = g(7)$
$= 7^2 - 4$
$= 45$
So, $g(f(2)) = 45$

c) $g(g(2))$

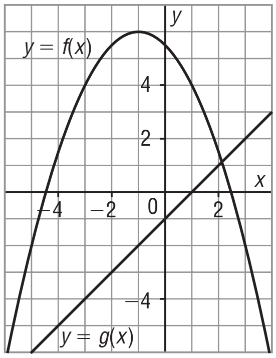
From part a, $g(2) = 0$
$g(g(2)) = g(0)$
$= 0^2 - 4$
$= -4$
So, $g(g(2)) = -4$

d) $f(f(2))$

From part b, $f(2) = 7$
$f(f(2)) = f(7)$
$= 3(7) + 1$
$= 22$
So, $f(f(2)) = 22$

B

7. Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below.



a) $f(g(4))$

From the graph of $y = g(x)$, $g(4) = 3$
From the graph of $y = f(x)$, $f(3) = -2$
So, $f(g(4)) = -2$

b) $g(f(3))$

From the graph of $y = f(x)$, $f(3) = -2$
From the graph of $y = g(x)$, $g(-2) = -3$
So, $g(f(3)) = -3$

8. Given the functions $f(x) = x^2 - 5x + 5$ and $g(x) = \frac{2x + 3}{x - 1}$, determine each value.

a) $f(g(-4))$

$$g(-4) = \frac{2(-4) + 3}{-4 - 1}$$
$$= 1$$
$$f(1) = 1^2 - 5(1) + 5$$
$$= 1$$
$$f(g(-4)) = 1$$

b) $g(f(2))$

$$f(2) = 2^2 - 5(2) + 5$$
$$= -1$$
$$g(-1) = \frac{2(-1) + 3}{-1 - 1}$$
$$= -0.5$$
$$g(f(2)) = -0.5$$

9. Given the functions $f(x) = |4 - x|$, $g(x) = (x - 4)^2$, and $h(x) = \sqrt{x}$, determine each value.

a) $f(g(1))$

$$g(1) = (1 - 4)^2$$
$$= 9$$
$$f(9) = |4 - 9|$$
$$= 5$$
$$f(g(1)) = 5$$

b) $h(g(-2))$

$$g(-2) = (-2 - 4)^2$$
$$= 36$$
$$h(36) = \sqrt{36}$$
$$= 6$$
$$h(g(-2)) = 6$$

c) $f(g(h(2)))$

$$h(2) = \sqrt{2}$$
$$g(\sqrt{2}) = (\sqrt{2} - 4)^2$$
$$= 2 - 8\sqrt{2} + 16$$
$$= 18 - 8\sqrt{2}$$
$$f(18 - 8\sqrt{2}) = |4 - 18 + 8\sqrt{2}|$$
$$= |-14 + 8\sqrt{2}|$$
$$= 14 - 8\sqrt{2}$$
$$f(g(h(2))) = 14 - 8\sqrt{2}$$

d) $h(g(f(2)))$

$$f(2) = |4 - 2|$$
$$= 2$$
$$g(2) = (2 - 4)^2$$
$$= 4$$
$$h(4) = \sqrt{4}$$
$$= 2$$
$$h(g(f(2))) = 2$$

10. Given $f(x) = 4x - 3$ and $g(x) = -2x^2 + 3x$, determine an explicit equation for each composite function, then state its domain and range.

a) $f(g(x))$

$f(g(x)) = f(-2x^2 + 3x)$
$f(g(x)) = 4(-2x^2 + 3x) - 3$
$f(g(x)) = -8x^2 + 12x - 3$
This is a quadratic function; its domain is: $x \in \mathbb{R}$
Use graphing technology to graph the function; its range is: $y \leq 1.5$

b) $g(f(x))$

$g(f(x)) = g(4x - 3)$
$g(f(x)) = -2(4x - 3)^2 + 3(4x - 3)$
$g(f(x)) = -32x^2 + 48x - 18 + 12x - 9$
$g(f(x)) = -32x^2 + 60x - 27$
This is a quadratic function; its domain is: $x \in \mathbb{R}$
From the graph of the function, its range is: $y \leq 1.125$

c) $g(g(x))$

$g(g(x)) = g(-2x^2 + 3x)$
$g(g(x)) = -2(-2x^2 + 3x)^2 + 3(-2x^2 + 3x)$
$= -8x^4 + 24x^3 - 18x^2 - 6x^2 + 9x$
$= -8x^4 + 24x^3 - 24x^2 + 9x$
This is a polynomial function; its domain is: $x \in \mathbb{R}$
Use graphing technology to graph the function; its range is: $y \leq 1.125$

d) $f(f(x))$

$f(f(x)) = f(4x - 3)$
$f(f(x)) = 4(4x - 3) - 3$
$f(f(x)) = 16x - 15$
This is a linear function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

11. Given $f(x) = x^3 - 5$ and $g(x) = 3x + 1$, determine an explicit equation for each composite function, then state its domain and range.

a) $f(g(x))$

$f(g(x)) = f(3x + 1)$
$f(g(x)) = (3x + 1)^3 - 5$
This is a cubic function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

b) $g(f(x))$

$g(f(x)) = g(x^3 - 5)$
$g(f(x)) = 3(x^3 - 5) + 1$
$g(f(x)) = 3x^3 - 14$
This is a cubic function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

c) $f(f(x))$

$f(f(x)) = f(x^3 - 5)$
$f(f(x)) = (x^3 - 5)^3 - 5$
This is a polynomial function with an odd degree; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

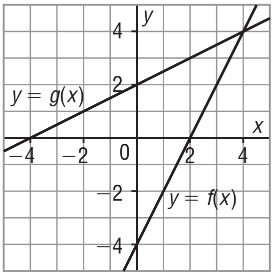
d) $g(g(x))$

$g(g(x)) = g(3x + 1)$
$g(g(x)) = 3(3x + 1) + 1$
$g(g(x)) = 9x + 4$
This is a linear function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

12. Can the composition of two linear functions form a quadratic function? Justify your answer.

No, two linear functions have the form $f(x) = mx + b$ and $g(x) = nx + c$. When I compose functions, I substitute one function for the variable in the other function, so for two linear functions, the composite function is also a linear function. For example, $f(g(x)) = m(nx + c) + b$, which simplifies to $f(g(x)) = mnx + mc + b$.

13. Given the graphs of $y = f(x)$ and $y = g(x)$



a) Determine $f(g(2))$ and $g(f(2))$.

From the graph:

$g(2) = 3$	$f(2) = 0$
$f(3) = 2$	$g(0) = 2$
So, $f(g(2)) = 2$	So, $g(f(2)) = 2$

b) Determine $f(g(1))$ and $g(f(1))$.

From the graph:

$g(1) = 2.5$	$f(1) = -2$
$f(2.5) = 1$	$g(-2) = 1$
So, $f(g(1)) = 1$	So, $g(f(1)) = 1$

c) How are the functions $f(x)$ and $g(x)$ related? Justify your answer.

From parts a and b, $f(g(2)) = g(f(2)) = 2$ and $f(g(1)) = g(f(1)) = 1$. The functions are inverses of each other. Their graphs are reflections of each other in the line $y = x$.

14. Use composition of functions to determine whether the functions in each pair are inverse functions.

a) $y = \frac{1}{3}x + 2$ and $y = 3x - 6$

Write $y = \frac{1}{3}x + 2$ as

$f(x) = \frac{1}{3}x + 2$, and write

$y = 3x - 6$ as $g(x) = 3x - 6$.

Determine:

$f(g(x)) = \frac{1}{3}(3x - 6) + 2$

$= x$

Determine:

$g(f(x)) = 3\left(\frac{1}{3}x + 2\right) - 6$

$= x$

Since $f(g(x)) = g(f(x)) = x$, the functions are inverses.

b) $y = 2x - 3$ and $y = 2x + 3$

Write $y = 2x - 3$ as $f(x) = 2x - 3$, and write $y = 2x + 3$ as $g(x) = 2x + 3$.

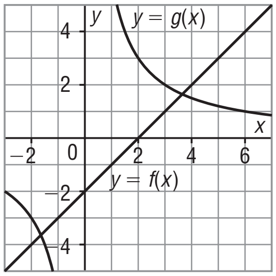
Determine:

$f(g(x)) = 2(2x + 3) - 3$

$= 4x + 3$

Since $f(g(x)) \neq x$, the functions are not inverses.

15. Given the graphs of $y = f(x)$ and $y = g(x)$



a) Determine the value of a for which $f(g(a)) = -1$.

Work backward.
Determine the value of x for which $f(x) = -1$.
From the graph, $x = 1$
Determine the value of x for which $g(x) = 1$.
From the graph, $x = 6$
So, for $f(g(a)) = -1$, $a = 6$

b) Determine the value of a for which $g(f(a)) = 2$.

Determine the value of x for which $g(x) = 2$.
From the graph, $x = 3$
Determine the value of x for which $f(x) = 3$.
From the graph, $x = 5$
So, for $g(f(a)) = 2$, $a = 5$

C

16. Given the functions $f(x) = x^2 - 2x$ and $g(x) = 3x + 2$, write an explicit expression for each value.

a) $g(f(a))$

Determine $g(f(x))$.
$g(f(x)) = 3(x^2 - 2x) + 2$
$g(f(x)) = 3x^2 - 6x + 2$
Substitute: $x = a$
$g(f(a)) = 3a^2 - 6a + 2$

b) $f(g(a))$

Determine $f(g(x))$.
$f(g(x)) = (3x + 2)^2 - 2(3x + 2)$
$f(g(x)) = 9x^2 + 12x + 4 - 6x - 4$
$f(g(x)) = 9x^2 + 6x$
Substitute: $x = a$
$f(g(a)) = 9a^2 + 6a$

c) $f(g(a - 1))$

From part b, $f(g(x)) = 9x^2 + 6x$
Substitute: $x = a - 1$
$f(g(a - 1)) = 9(a - 1)^2 + 6(a - 1)$
$f(g(a - 1)) = 9a^2 - 18a + 9 + 6a - 6$
$f(g(a - 1)) = 9a^2 - 12a + 3$

d) $f(g(1 - a))$

From part b, $f(g(x)) = 9x^2 + 6x$
Substitute: $x = 1 - a$
$f(g(1 - a)) = 9(1 - a)^2 + 6(1 - a)$
$f(g(1 - a)) = 9 - 18a + 9a^2 + 6 - 6a$
$f(g(1 - a)) = 15 - 24a + 9a^2$

17. Given the functions $f(x) = x^2 - 2x + 2$, $g(x) = 5x - 2$, and $h(x) = \sqrt{x + 3}$

a) Determine the value of a for which $g(h(a)) = 13$.

Write an explicit equation for $g(h(x))$.

$g(h(x)) = 5(\sqrt{x + 3}) - 2$

$g(h(x)) = 5\sqrt{x + 3} - 2$ Substitute: $x = a$, $g(h(a)) = 13$

$13 = 5\sqrt{a + 3} - 2$

$5\sqrt{a + 3} = 15$

$\sqrt{a + 3} = 3$

$a + 3 = 9$

$a = 6$

Verify the solution.

b) Determine the values of a for which $f(g(a)) = 5$.

Write an explicit equation for $f(g(x))$.

$f(g(x)) = (5x - 2)^2 - 2(5x - 2) + 2$

$f(g(x)) = 25x^2 - 20x + 4 - 10x + 4 + 2$

$f(g(x)) = 25x^2 - 30x + 10$ Substitute: $x = a$, $f(g(a)) = 5$

$5 = 25a^2 - 30a + 10$

$0 = 25a^2 - 30a + 5$ Factor.

$0 = 5(5a^2 - 6a + 1)$

$0 = 5(5a - 1)(a - 1)$

$a = 0.2$ or $a = 1$

c) Why are there two values of a for part b but only one value for part a?

In part a, $g(h(x))$ is a radical function and its related equation has only one solution. In part b, $f(g(x))$ is a quadratic function and its related equation has two solutions.