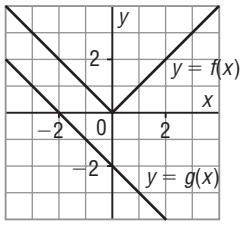
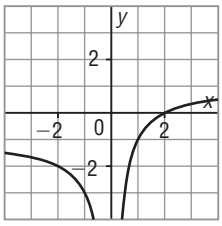
Checkpoint: Assess Your Understanding, pages 287–290

4.1

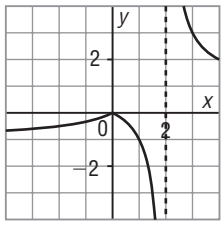
1. Multiple Choice Given the graphs of $y = f(x)$ and $y = g(x)$, which graph below represents $y = \dfrac{f(x)}{g(x)}$?



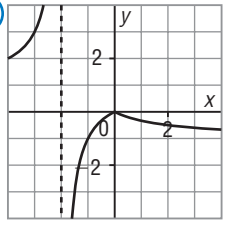
A.



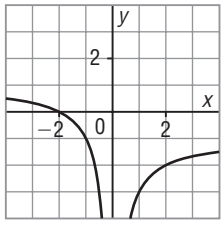
B.



C.

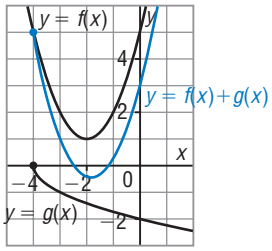


D.

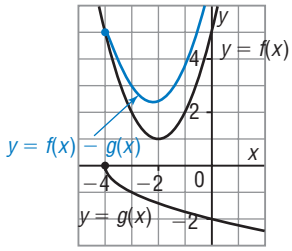


2. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each given function. Identify its domain and range; approximate the range where necessary.

a) $y = f(x) + g(x)$



b) $y = f(x) - g(x)$



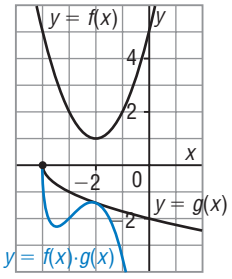
From the graphs:

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$	$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)}$
−4	5	0	5	5	0	undefined
−3	2	−1	1	3	−2	−2
−2	1	$\doteq -1.4$	$\doteq -0.4$	$\doteq 2.4$	$\doteq -1.4$	$\doteq -0.7$
−1	2	$\doteq -1.7$	$\doteq 0.3$	$\doteq 3.7$	$\doteq -3.5$	$\doteq -1.2$
0	5	−2	3	7	−10	−2.5

Plot points at: (−4, 5), (−3, 1), (−2, −0.4), (−1, 0.3), (0, 3)
Join the points with a smooth curve.
Domain: $x \geq -4$
Approximate range: $y \geq -0.4$

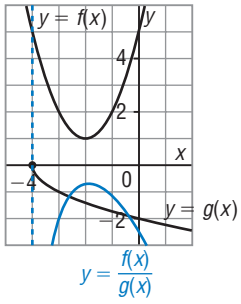
Plot points at: (−4, 5), (−3, 3), (−2, 2.4), (−1, 3.7)
Join the points with a smooth curve.
Domain: $x \geq -4$
Approximate range: $y \geq 2.4$

c) $y = f(x) \cdot g(x)$



Plot points at: (−4, 0), (−3, −2), (−2, −1.4), (−1, −3.5)
Join the points with a smooth curve.
Domain: $x \geq -4$
Range: $y \leq 0$

d) $y = \dfrac{f(x)}{g(x)}$



Plot points at: (−3, −2), (−2, −0.7), (−1, −1.2), (0, −2.5)
Since $g(-4) = 0$, draw an asymptote at $x = -4$.
Join the points with a smooth curve.
Domain: $x > -4$
Approximate range: $y \leq -0.7$

4.2

3. Multiple Choice Given $f(x) = x - 2$ and $g(x) = \sqrt{x}$, what is the domain of $h(x) = f(x) \cdot g(x)$?

A. $x \in \mathbb{R}$ **B.** $x \neq 2$ **C.** $x > 2$ **D.** $x \geq 0$

4. Use $f(x) = x^2 + x - 20$.

a) Write explicit equations for two functions $g(x)$ and $k(x)$ so that $f(x) = g(x) \cdot k(x)$.

Sample response:
Factor: $f(x) = (x + 5)(x - 4)$
So, $g(x) = x + 5$ and $k(x) = x - 4$

b) Write explicit equations for three functions $g(x)$, $h(x)$, and $k(x)$ so that $f(x) = g(x) - h(x) - k(x)$.

Sample response:
$f(x) = x^2 + x - 20$
$f(x) = x^2 - (-x) - 20$
So, $g(x) = x^2$; $h(x) = -x$; and $k(x) = 20$

c) Write explicit equations for two functions $g(x)$ and $k(x)$ so that $f(x) = \frac{g(x)}{k(x)}$.

Sample response:
Multiply and divide $x^2 + x - 20$ by a non-zero expression.

$$f(x) = \frac{(x^2 + x - 20)(x^2 + 4)}{x^2 + 4}$$

So, $g(x) = (x^2 + x - 20)(x^2 + 4)$ and $k(x) = x^2 + 4$

5. Use $f(x) = 3x^2 - 1$, $g(x) = \frac{1}{x + 2}$, and $h(x) = \sqrt{x - 5}$.

i) Write an explicit equation for each function below.

ii) State the domain and range of each function; approximate the range where necessary.

a) $h(x) = f(x) + g(x)$

i) $h(x) = 3x^2 - 1 + \frac{1}{x + 2}$

ii) The domain is: $x \neq -2$
Use technology; the range is: $y \in \mathbb{R}$

b) $d(x) = g(x) - h(x)$

i) $d(x) = \frac{1}{x + 2} - \sqrt{x - 5}$

ii) The domain is: $x \geq 5$
Use technology; the range is: $y \leq \frac{1}{7}$

c) $p(x) = f(x) \cdot g(x)$

i) $p(x) = (3x^2 - 1)\left(\frac{1}{x + 2}\right)$

$p(x) = \frac{3x^2 - 1}{x + 2}$

ii) The domain is: $x \neq -2$
Use technology; the range is approximately: $y \geq -0.5$ or $y \leq -23.5$

d) $q(x) = \frac{h(x)}{g(x)}$

i) $q(x) = \frac{\sqrt{x - 5}}{\frac{1}{x + 2}}$

$q(x) = (x + 2)\sqrt{x - 5}$

ii) The domain is: $x \geq 5$
Use technology; the range is: $y \geq 0$