Lesson 5.5 Exercises, pages 393–398

A

4. Simplify each expression. Use a calculator to verify the answer.

a) $\log 6 + \log 5$

$= \log (6 \cdot 5)$
$= \log 30$
Verify: $\log 6 + \log 5 = 1.4771...$
$\log 30 = 1.4771...$

b) $3 \log 2$

$= \log 2^3$
$= \log 8$
Verify: $3 \log 2 = 0.9030...$
$\log 8 = 0.9030...$

c) $\log 48 - \log 6$

$= \log \left(\frac{48}{6}\right)$
$= \log 8$
Verify: $\log 48 - \log 6 = 0.9030...$
$\log 8 = 0.9030...$

d) $\log 8 + \log 5$

$= \log (8 \cdot 5)$
$= \log 40$
Verify: $\log 8 + \log 5 = 1.6020...$
$\log 40 = 1.6020...$

5. Write each expression as a single logarithm.

a) $\log a - \log b$

$= \log\left(\frac{a}{b}\right)$

b) $\log x + \log y$

$= \log xy$

c) $5 \log m$

$= \log m^5$

d) $\log x - \log y + \log z$

$= \log\left(\frac{x}{y}\right) + \log z$

$= \log\left(\frac{xz}{y}\right)$

B

6. Use each law of logarithms to write an expression that is equal to log 16. Use a calculator to verify each expression.

a) the product law

Sample response: Determine 2 numbers whose product is 16: 2 and 8

So, $\log 16 = \log 2 + \log 8$

Verify: $\log 16 = 1.2041\ldots$ $\quad \log 2 + \log 8 = 1.2041\ldots$

b) the quotient law

Determine 2 numbers whose quotient is 16: 80 and 5

So, $\log 16 = \log 80 - \log 5$

Verify: $\log 16 = 1.2041\ldots$ $\quad \log 80 - \log 5 = 1.2041\ldots$

c) the power law

Write 16 as a power: 4^2

So, $\log 16 = 2 \log 4$

Verify: $\log 16 = 1.2041\ldots$ $\quad 2 \log 4 = 1.2041\ldots$

7. Substitute values of a and b to verify each statement.

a) $\frac{\log a}{\log b} \neq \log\left(\frac{a}{b}\right)$

Substitute: $a = 3$ and $b = 5$

$\frac{\log a}{\log b} = \frac{\log 3}{\log 5}$

$= 0.6826\ldots$

$\log\left(\frac{a}{b}\right) = \log\left(\frac{3}{5}\right)$

$= -0.2218\ldots$

Since the left side is not equal to the right side, the statement is verified.

b) $\log (a + b) \neq \log ab$

Substitute: $a = 3$ and $b = 5$

$\log (a + b) = \log 8$

$= 0.9030\ldots$

$\log ab = \log 15$

$= 1.1760\ldots$

Since the left side is not equal to the right side, the statement is verified.

8. Write each expression as a single logarithm.

a) $\log x - 5\log y$

$= \log x - \log y^5$

$= \log\left(\frac{x}{y^5}\right)$

b) $\frac{1}{2}\log x + 3\log y$

$= \log x^{\frac{1}{2}} + \log y^3$

$= \log\left(x^{\frac{1}{2}}y^3\right)$

c) $\frac{2}{3}\log_5 x - 4\log_5 y - 3\log_5 z$

$= \log_5 x^{\frac{2}{3}} - \log_5 y^4 - \log_5 z^3$

$= \log_5 x^{\frac{2}{3}} - (\log_5 y^4 + \log_5 z^3)$

$= \log_5 x^{\frac{2}{3}} - \log_5 y^4 z^3$

$= \log_5\left(\frac{x^{\frac{2}{3}}}{y^4 z^3}\right)$

d) $5 + \log_2 x$

$= \log_2 2^5 + \log_2 x$

$= \log_2 32 + \log_2 x$

$= \log_2 32x$

9. Explain each step in this proof of the power law for logarithms.
To prove that $\log_b x^k = k\log_b x$:

Let: $\log_b x = n$	Write the logarithm as a power.
Then $x = b^n$	Raise each side to the power k.
$x^k = (b^n)^k$	Simplify.
$x^k = b^{kn}$	Take the logarithm base b of each side.
$\log_b x^k = \log_b b^{kn}$	Simplify the right side.
$\log_b x^k = kn$	Substitute: $n = \log_b x$
$\log_b x^k = k\log_b x$	

10. Use the strategy from the proof of the product law for logarithms to prove the quotient law: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Let $\log_b x = m$ and $\log_b y = n$	Apply the definition of a logarithm.
Then $x = b^m$ $\quad y = b^n$	
So, $\frac{x}{y} = \frac{b^m}{b^n}$	Use the quotient law for exponents.
$\frac{x}{y} = b^{m-n}$	Write this exponential statement as a logarithmic statement.
$\log_b\left(\frac{x}{y}\right) = m - n$	Substitute for m and n.
$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	

11. Use two different strategies to write $2(\log x + \log y)$ as a single logarithm.

Strategy 1
$2(\log x + \log y)$
$= 2 \log x + 2 \log y$
$= \log x^2 + \log y^2$
$= \log x^2y^2$

Strategy 2
$2(\log x + \log y)$
$= 2 \log xy$
$= \log (xy)^2$
$= \log x^2y^2$

12. Write each expression as a single logarithm.

a) $3 \log 2 + \log 6$

$= \log 2^3 + \log 6$
$= \log 8 + \log 6$
$= \log (8 \cdot 6)$
$= \log 48$

b) $\frac{1}{2} \log 9 + 2 \log 5$

$= \log 9^{\frac{1}{2}} + \log 5^2$
$= \log 3 + \log 25$
$= \log 75$

c) $3 \log_2 6 - 2$

$= \log_2 6^3 - \log_2 2^2$
$= \log_2 216 - \log_2 4$
$= \log_2 \left(\frac{216}{4}\right)$
$= \log_2 54$

d) $5 \log_5 2 - \log_5 4 + 2$

$= \log_5 2^5 - \log_5 4 + \log_5 5^2$
$= \log_5 32 - \log_5 4 + \log_5 25$
$= \log_5 \left(\frac{32 \cdot 25}{4}\right)$
$= \log_5 200$

13. Evaluate each expression.

a) $2 \log_3 6 - 3 \log_3 2 + \log_3 18$

$= \log_3 6^2 - \log_3 2^3 + \log_3 18$
$= \log_3 36 - \log_3 8 + \log_3 18$
$= \log_3 \left(\frac{36 \cdot 18}{8}\right)$
$= \log_3 81$
$= \log_3 3^4$
$= 4$

b) $\frac{1}{2} \log_2 36 + \log_2 12 - 2 \log_2 3$

$= \log_2 36^{\frac{1}{2}} + \log_2 12 - \log_2 3^2$
$= \log_2 6 + \log_2 12 - \log_2 9$
$= \log_2 \left(\frac{6 \cdot 12}{9}\right)$
$= \log_2 8$
$= \log_2 2^3$
$= 3$

c) $9 \log_9 3 - \log_9 75 + 2 \log_9 5$

$= \log_9 3^9 - \log_9 75 + \log_9 5^2$
$= \log_9 19\,683 - \log_9 75 + \log_9 25$
$= \log_9 \left(\frac{19\,683 \cdot 25}{75}\right)$
$= \log_9 6561$
$= \log_9 9^4$
$= 4$

d) $\log_4 98 - 2 \log_4 7 - 2$

$= \log_4 98 - \log_4 7^2 - 2$
$= \log_4 98 - \log_4 49 - 2$
$= \log_4 \left(\frac{98}{49}\right) - 2$
$= \log_4 2 - 2$
$= \log_4 4^{\frac{1}{2}} - 2$
$= \frac{1}{2} - 2$
$= -\frac{3}{2}$

14. Given $\log a \doteq 1.301$, determine an approximate value for each logarithm.

a) $\log a^3$

$= 3 \log a$
$\doteq 3(1.301)$
$\doteq 3.903$

b) $\log 10a$

$= \log 10 + \log a$
$\doteq 1 + 1.301$
$\doteq 2.301$

c) $\log \left(\frac{a^2}{100}\right)$

$= \log a^2 - \log 100$
$= 2 \log a - 2$
$\doteq 2(1.301) - 2$
$\doteq 0.602$

15. Identify the errors in the solution to the question below. Write the correct solution.

Write $\log \left(\frac{a^{\frac{1}{2}}}{c^3b^2}\right)$ in terms of log *a*, log *b*, and log *c*.

$\log \left(\frac{a^{\frac{1}{2}}}{c^3b^2}\right)$
$= \log a^{\frac{1}{2}} - \log c^3b^2$
$= \log a^{\frac{1}{2}} - \log c^3 + \log b^2$
$= \frac{1}{2} \log a - \log 3c + 2 \log b$

$\log \left(\frac{a^{\frac{1}{2}}}{c^3b^2}\right)$
$= \log a^{\frac{1}{2}} - \log c^3b^2$
$= \log a^{\frac{1}{2}} - (\log c^3 + \log b^2)$
$= \log a^{\frac{1}{2}} - \log c^3 - \log b^2$
$= \frac{1}{2} \log a - 3 \log c - 2 \log b$

In the second line of the solution, when $\log c^3b^2$ is written as a sum of logarithms, both logarithms should be negative. In the third line of the solution, $\log c^3$ should be $3 \log c$.

16. Write each expression in terms of log *a*, log *b*, and/or log *c*.

a) $\log a^3b^{\frac{1}{2}}$

$= \log a^3 + \log b^{\frac{1}{2}}$
$= 3 \log a + \frac{1}{2} \log b$

b) $\log ab^2c^{\frac{2}{3}}$

$= \log a + \log b^2 + \log c^{\frac{2}{3}}$
$= \log a + 2 \log b + \frac{2}{3} \log c$

c) $\log \left(\frac{a^3}{b^2}\right)$

$= \log a^3 - \log b^2$
$= 3 \log a - 2 \log b$

d) $\log \left(\frac{a^4b^{\frac{3}{5}}}{c}\right)$

$= \log a^4 + \log b^{\frac{3}{5}} - \log c$
$= 4 \log a + \frac{3}{5} \log b - \log c$

17. Given $\log 3 \doteq 0.477$ and $\log 7 \doteq 0.845$, determine the approximate value of $\log (132\ 300)$ without using a calculator.

Write 132 300 as a product of factors that involve powers of 3 and 7.

$132\ 300 \div 9 = 14\ 700$

$14\ 700 \div 3 = 4900$

$4900 \div 49 = 100$

So, $132\ 300 = 3^3 \cdot 7^2 \cdot 10^2$

$$\begin{aligned}\log (132\ 300) &= \log (3^3 \cdot 7^2 \cdot 10^2)\\ &= \log 3^3 + \log 7^2 + \log 10^2\\ &= 3 \log 3 + 2 \log 7 + 2\\ &\doteq 3(0.477) + 2(0.845) + 2\\ &\doteq 5.121\end{aligned}$$

C

18. Write each expression as a single logarithm.

a) $3 \log x + \log (2x - 3)$

$= \log x^3 + \log (2x - 3)$

$= \log x^3(2x - 3)$

b) $\log (x + 1) + \log (2x - 1)$

$= \log (x + 1)(2x - 1)$

c) $\log (x^2 - 1) - \log (x - 1)$

$= \log \left(\frac{x^2 - 1}{x - 1}\right)$

$= \log \frac{(x - 1)(x + 1)}{x - 1}$

$= \log (x + 1)$

d) $\log (2x^2 + x - 3) - \log (x^2 - 1)$

$= \log \left(\frac{2x^2 + x - 3}{x^2 - 1}\right)$

$= \log \frac{(2x + 3)(x - 1)}{(x - 1)(x + 1)}$

$= \log \left(\frac{2x + 3}{x + 1}\right)$

19. Without using the power law, prove the law of logarithms for radicals:
$\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x,\ b > 0,\ b \neq 1,\ k \in \mathbb{N},\ x > 0$

Sample response:

Let: $\log_b x = n$	Write the logarithm as a power.
Then $x = b^n$	Raise each side to the power $\frac{1}{k}$.
$x^{\frac{1}{k}} = b^{\frac{n}{k}}$	Take the logarithm base b of each side.
$\log_b x^{\frac{1}{k}} = \log_b b^{\frac{n}{k}}$	Simplify the right side.
$\log_b x^{\frac{1}{k}} = \frac{n}{k}$	Substitute: $n = \log_b x$
$\log_b x^{\frac{1}{k}} = \frac{1}{k} \log_b x$	
Or, $\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x$	