Lesson 5.8 Exercises, pages 435–439

A

3. Use the equation $200 = 100(1.05)^t$ to determine the time in years it will take an investment of $100 to double when it is invested in an account that pays 5% annual interest, compounded annually.

$200 = 100(1.05)^t$ Simplify.
$2 = 1.05^t$ Take the common logarithm of each side.
$\log 2 = \log 1.05^t$
$\log 2 = t \log 1.05$
$t = \frac{\log 2}{\log 1.05}$
$t = 14.2066...$

It will take approximately 14 years for the investment to double.

B

4. In 1949, Vancouver Island experienced an earthquake with a magnitude of 8.1. How many times as intense as the 5.0-magnitude Ontario-Quebec earthquake in 2010 was the Vancouver Island earthquake? Give the answer to the nearest whole number.

Use: $M = \log\left(\frac{I}{S}\right)$

For Vancouver Island:
Substitute: $M = 8.1$
$8.1 = \log\left(\frac{I}{S}\right)$
$\frac{I}{S} = 10^{8.1}$
$I = 10^{8.1}S$

For Ontario-Quebec:
Substitute: $M = 5.0$
$5.0 = \log\left(\frac{I}{S}\right)$
$\frac{I}{S} = 10^{5.0}$
$I = 10^{5.0}S$

$$\frac{\text{the intensity of the Vancouver Island earthquake}}{\text{the intensity of the Ontario-Quebec earthquake}} = \frac{10^{8.1}S}{10^{5.0}S}$$
$$= 10^{3.1}$$
$$= 1258.9254...$$

The earthquake in Vancouver Island was approximately 1259 times as intense as the earthquake in Ontario-Quebec.

5. Why is the intensity of an earthquake with magnitude 6 not twice the intensity of an earthquake with magnitude 3?

The intensities of earthquakes are measured on a logarithmic scale, which is not linear. The intensity of an earthquake with magnitude 6 is 10^6S.
The intensity of an earthquake with magnitude 3 is 10^3S.
So, the intensity of the earthquake with magnitude 6 is 10^{6-3}, or 10^3 times as great as the intensity of the earthquake with magnitude 3.

6. A student is saving money to buy a used car. The student deposits \$150 monthly in a savings account that pays 3% annual interest, compounded monthly.

a) How long will it take the student to save \$5000?

Use: $FV = \dfrac{R[(1 + i)^n - 1]}{i}$

Substitute: $FV = 5000$; $R = 150$; $i = \dfrac{0.03}{12}$, or 0.0025

$$5000 = \frac{150[(1 + 0.0025)^n - 1]}{0.0025}$$

$$\left(\frac{5000}{150}\right)(0.0025) = 1.0025^n - 1$$

$$\frac{1}{12} = 1.0025^n - 1$$

$$\frac{13}{12} = 1.0025^n$$

$$\log\left(\frac{13}{12}\right) = \log 1.0025^n$$

$$\log\left(\frac{13}{12}\right) = n \log 1.0025$$

$$n = \frac{\log\left(\frac{13}{12}\right)}{\log 1.0025}$$

$$n = 32.0570\ldots$$

It will take the student approximately 32 months or 2 years 8 months to save the money.

b) How much money did the student deposit in the savings account?

The student deposited: 32(\$150) = \$4800

7. A student borrows \$5000 to buy a used car. The loan payments are \$150 a month at 9% annual interest, compounded monthly.

a) How long will it take the student to repay the loan?

Use the formula: $PV = \dfrac{R[1 - (1 + i)^{-n}]}{i}$

Substitute: $PV = 5000$, $R = 150$, $i = \dfrac{0.09}{12}$, or 0.0075

$$5000 = \frac{150[1 - (1 + 0.0075)^{-n}]}{0.0075}$$

$$\left(\frac{5000}{150}\right)(0.0075) = 1 - 1.0075^{-n}$$

$$0.25 = 1 - 1.0075^{-n}$$

$$1.0075^{-n} = 0.75$$

$$\log 1.0075^{-n} = \log 0.75$$

$$-n \log 1.0075 = \log 0.75$$

$$n = \frac{\log 0.75}{-\log 1.0075}$$

$$n = 38.5012\ldots$$

It will take the student approximately 39 months, or 3.25 years to repay the loan.

b) How much money did the student pay?

The student paid approximately: 38.5($150) = $5775

8. Look at the answers to questions 6 and 7. Which may be the better way to finance the purchase of a car? Explain.

It is better to save to buy a car rather than to borrow money to buy the car. The person who saved the money spent $4800. The person who borrowed the money spent $5775.

9. The acidity or alkalinity of a solution is measured using a logarithmic scale called the *pH scale*. A solution that has a pH of 7 is neutral. For each increase of 1 pH, a solution is 10 times as alkaline. For each decrease of 1 pH, a solution is 10 times as acidic.

a) A sample of soda water has a pH of 3.8. A sample of vinegar has a pH of 2.8.

i) Which sample is more acidic?

The lesser the pH, the more acidic a solution is.
So, vinegar is more acidic than soda water.

ii) How many times as acidic is the sample?

The difference in pH is: $3.8 - 2.8 = 1$
A decrease of 1 in pH represents 10 times the acidity.
So, vinegar is 10 times as acidic as soda water.

b) A sample of household ammonia has a pH of 11.5. A sample of sea water has a pH of 8.4.

i) Which sample is more alkaline?

The greater the pH, the more alkaline a solution is. So, household ammonia is more alkaline than sea water.

ii) How many times as alkaline is the sample? Give the answer to the nearest whole number.

The difference in pH is: $11.5 - 8.4 = 3.1$
An increase of 1 in pH represents 10 times the alkalinity.
So, household ammonia is $10^{3.1}$, or approximately 1259 times as alkaline as sea water.

10. The *decibel scale* measures the intensity of sound. The loudness of a sound, L decibels (dB), can be determined using the function $L = 10 \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is the intensity of the quietest sound that can be detected.

a) The loudness of normal conversation is 60 dB. Calculate the intensity of this sound in terms of I_0.

Use: $L = 10 \log\left(\frac{I}{I_0}\right)$ Substitute: $L = 60$

$60 = 10 \log\left(\frac{I}{I_0}\right)$ Simplify.

$6 = \log\left(\frac{I}{I_0}\right)$ Write as an exponential statement.

$\frac{I}{I_0} = 10^6$

$I = 10^6 I_0$

b) The loudness of a rock concert is 120 dB. Calculate the intensity of this sound in terms of I_0.

Use: $L = 10 \log\left(\frac{I}{I_0}\right)$ Substitute: $L = 120$

$120 = 10 \log\left(\frac{I}{I_0}\right)$ Simplify.

$12 = \log\left(\frac{I}{I_0}\right)$ Write as an exponential statement.

$\frac{I}{I_0} = 10^{12}$

$I = 10^{12} I_0$

c) How many times as intense as the sound of normal conversation is the sound of a rock concert?

The intensity of the sound of normal conversation is: $10^6 I_0$
The intensity of the sound of a rock concert is: $10^{12} I_0$
So, the sound of a rock concert is $\frac{10^{12}}{10^6}$, or 10^6 times as intense as normal conversation.

11. The loudness of city traffic is 80 dB and the loudness of a car horn is 110 dB. Use the formula in question 10. How many times as intense as the sound of city traffic is the sound of a car horn?

The intensity of the sound of city traffic is: $10^8 I_0$
The intensity of the sound of a car horn is: $10^{11} I_0$
So, the sound of a car horn is $\frac{10^{11}}{10^8}$, or 10^3 times as intense as the sound of city traffic.

C

12. Each of two people has a mortgage of $200 000 with an annual interest rate of 3.5%. Person A makes payments of $500.00 every two weeks, and the interest is compounded every two weeks. Person B makes monthly payments of $1000, and the interest is compounded monthly. Who pays off the mortgage first? How much sooner is it paid?

For person A

Use: $PV = \dfrac{R[1 - (1 + i)^{-n}]}{i}$

Substitute:

$PV = 200\ 000$, $R = 500$, $i = \dfrac{0.035}{26}$

$$200\ 000 = \frac{500\left[1 - \left(1 + \frac{0.035}{26}\right)^{-n}\right]}{\frac{0.035}{26}}$$

$$\frac{7}{13} = 1 - \left(1 + \frac{0.035}{26}\right)^{-n}$$

$$\left(1 + \frac{0.035}{26}\right)^{-n} = \frac{6}{13}$$

$$\log\left(1 + \frac{0.035}{26}\right)^{-n} = \log\left(\frac{6}{13}\right)$$

$$-n\log\left(1 + \frac{0.035}{26}\right) = \log\left(\frac{6}{13}\right)$$

$$n = \frac{\log\left(\frac{6}{13}\right)}{-\log\left(1 + \frac{0.035}{26}\right)}$$

$n = 574.7561\ldots$

There will be approximately 575 payments. So, the time until the mortgage is paid is:

$\dfrac{575}{26}$ years $\doteq 22.1$ years

For person B

Use: $PV = \dfrac{R[1 - (1 + i)^{-n}]}{i}$

Substitute:

$PV = 200\ 000$, $R = 1000$, $i = \dfrac{0.035}{12}$

$$200\ 000 = \frac{1000\left[1 - \left(1 + \frac{0.035}{12}\right)^{-n}\right]}{\frac{0.035}{12}}$$

$$\frac{1.75}{3} = 1 - \left(1 + \frac{0.035}{12}\right)^{-n}$$

$$\left(1 + \frac{0.035}{12}\right)^{-n} = \frac{1.25}{3}$$

$$\log\left(1 + \frac{0.035}{12}\right)^{-n} = \log\left(\frac{1.25}{3}\right)$$

$$n = \frac{\log\left(\frac{1.25}{3}\right)}{-\log\left(1 + \frac{0.035}{12}\right)}$$

$n = 300.5982\ldots$

There will be approximately 301 payments.

So, the time until the mortgage is paid is:

$\dfrac{301}{12}$ years $\doteq 25.1$ years

Person A pays off the mortgage 3 years earlier than person B.