

Lesson 8.2 Exercises, pages 701–705

A

3. Evaluate each factorial.

a) $4!$

$$= 4 \cdot 3 \cdot 2 \cdot 1$$
$$= 24$$

b) $1!$

$$= 1$$

c) $5!$

$$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$= 120$$

d) $0!$

$$= 1$$

4. Determine each value.

a) ${}_7P_2$

$$\begin{aligned} &= \frac{7!}{(7-2)!} \\ &= \frac{7!}{5!} \\ &= \frac{7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\ &= 42 \end{aligned}$$

b) ${}_5P_5$

$$\begin{aligned} &= \frac{5!}{(5-5)!} \\ &= \frac{5!}{0!} \\ &= \frac{5!}{1} \\ &= 120 \end{aligned}$$

c) ${}_8P_7$

$$\begin{aligned} &= \frac{8!}{(8-7)!} \\ &= \frac{8!}{1!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} \\ &= 40\,320 \end{aligned}$$

d) ${}_6P_1$

$$\begin{aligned} &= \frac{6!}{(6-1)!} \\ &= \frac{6!}{5!} \\ &= \frac{6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\ &= 6 \end{aligned}$$

5. a) Use a graphic organizer to determine the number of ways to arrange the letters in each word.

i) ELK

Make an organized list:
ELK; EKL; KEL; KLE; LEK; LKE
The letters can be arranged
in 6 ways.

ii) LYNX

Make an organized list:
LYNX; LYXN; LNYX; LNXY;
LXYN; LXNY; YNXL; YNLX;
YXNL; YXLN; YLNX; YLXN;
NXLY; NXYL; NLXY; NLYX;
NYLX; NYXL; XLYN; XLNY;
XNLY; XNYL; XYNL; XYLN
The letters can be arranged in 24 ways.

b) Use factorial notation to determine the number of ways to arrange the letters in each word.

i) BISON

The number of permutations
is: $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, or $5!$
 $5! = 120$
The letters can be arranged
in 120 ways.

ii) FALCON

The number of permutations is:
 $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, or $6!$
 $6! = 720$
The letters can be
arranged in 720 ways.

B

6. a) How many 2-letter permutations are there for each word in question 5?

$$\begin{aligned} \text{ELK: } {}_3P_2 &= \frac{3!}{(3-2)!} \\ &= \frac{3!}{1!} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{LYNX: } {}_4P_2 &= \frac{4!}{(4-2)!} \\ &= \frac{4!}{2!} \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{BISON: } {}_5P_2 &= \frac{5!}{(5-2)!} \\ &= \frac{5!}{3!} \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{FALCON: } {}_6P_2 &= \frac{6!}{(6-2)!} \\ &= \frac{6!}{4!} \\ &= 30 \end{aligned}$$

- b) How many 3-letter permutations are there for each word in question 5?

$$\begin{aligned} \text{ELK: } {}_3P_3 &= \frac{3!}{(3-3)!} \\ &= \frac{3!}{0!} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{LYNX: } {}_4P_3 &= \frac{4!}{(4-3)!} \\ &= \frac{4!}{1!} \\ &= 24 \end{aligned}$$

$$\begin{aligned} \text{BISON: } {}_5P_3 &= \frac{5!}{(5-3)!} \\ &= \frac{5!}{2!} \\ &= 60 \end{aligned}$$

$$\begin{aligned} \text{FALCON: } {}_6P_3 &= \frac{6!}{(6-3)!} \\ &= \frac{6!}{3!} \\ &= 120 \end{aligned}$$

- c) Describe any patterns you notice.

When the word has more than 3 letters, the number of 3-letter permutations is the number of 2-letter permutations multiplied by 2 less than the number of letters in the word. When the word has 3 letters, the numbers of 2-letter and 3-letter permutations are the same.

7. The music teacher must arrange 5 tunes for the senior jazz band to perform at Music Night. She has 20 tunes to choose from.

- a) How many arrangements are possible?

Use the formula: ${}_nP_r = \frac{n!}{(n-r)!}$ Substitute: $n = 20, r = 5$

$$\begin{aligned} {}_{20}P_5 &= \frac{20!}{(20-5)!} \\ &= \frac{20!}{15!} \\ &= 1\,860\,480 \end{aligned}$$

There are 1 860 480 possible arrangements.

- b) What other strategy could you use to determine the number of arrangements?

Use the counting principle. There are 20 ways to choose the 1st tune; 19 ways to choose the 2nd tune; and so on. So, the number of possible arrangements is: $20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 = 1\,860\,480$

8. In the World Cup of soccer, 32 teams compete for the title. What is the number of ways that the winner, runners-up, third, and fourth place prizes could be awarded? Verify your answer.

Use the formula: ${}_n P_r = \frac{n!}{(n-r)!}$ Substitute: $n = 32, r = 4$

$$\begin{aligned} {}_{32} P_4 &= \frac{32!}{(32-4)!} \\ &= \frac{32!}{28!} \\ &= 863\,040 \end{aligned}$$

There are 863 040 ways that the prizes could be awarded.

Verify:

Use the equation ${}_n P_r = \frac{n!}{(n-r)!}$.

Substitute ${}_n P_r = 863\,040$ and $r = 4$, then solve for n .

$$\begin{aligned} 863\,040 &= \frac{n!}{(n-4)!} \\ 863\,040 &= n(n-1)(n-2)(n-3) \end{aligned}$$

$$\sqrt[4]{863\,040} \doteq 30$$

So, try 4 consecutive numbers with 30 as one of the middle numbers:

$$32 \cdot 31 \cdot 30 \cdot 29 = 863\,040$$

So, $n = 32$

Since there are 32 teams, the solution is correct.

9. The longest English non-technical words with no repeated letters are *dermatoglyphics*, *misconjugatedly*, and *uncopyrightable*. What is the total number of ways to arrange all the letters in each word?

Each word has 15 letters.

There are 15! ways to arrange the letters.

$$15! = 1.3076 \dots \times 10^{12}$$

There are about 1.31×10^{12} ways to arrange all the letters in each word.

10. Solve each equation for n or r .

a) ${}_n P_2 = 90$

$${}_n P_2 = \frac{n!}{(n-2)!}$$

$$90 = \frac{n!}{(n-2)!}$$

$$90 = n(n-1)$$

$$0 = n^2 - n - 90$$

$$0 = (n-10)(n+9)$$

$$n = 10 \text{ or } n = -9$$

Since n cannot be negative,

$$n = 10$$

b) ${}_n P_3 = 120$

$${}_n P_3 = \frac{n!}{(n-3)!}$$

$$120 = \frac{n!}{(n-3)!}$$

$$120 = n(n-1)(n-2)$$

Use a calculator.

$$\sqrt[3]{120} \doteq 4.9$$

So, try 3 consecutive numbers with 5 as the middle number:

$$6 \cdot 5 \cdot 4 = 120$$

$$\text{So, } n = 6$$

$$\text{c) } {}_9P_r = 72$$

$${}_9P_r = \frac{9!}{(9-r)!}$$

$$72 = \frac{9!}{(9-r)!}$$

$$72 = \frac{362\,880}{(9-r)!}$$

$$(9-r)! = \frac{362\,880}{72}$$

$$(9-r)! = 5040$$

Since $7! = 5040$, then

$$9-r = 7$$

$$r = 2$$

$$\text{d) } {}_7P_r = 210$$

$${}_7P_r = \frac{7!}{(7-r)!}$$

$$210 = \frac{7!}{(7-r)!}$$

$$210 = \frac{5040}{(7-r)!}$$

$$(7-r)! = \frac{5040}{210}$$

$$(7-r)! = 24$$

Since $4! = 24$, then

$$7-r = 4$$

$$r = 3$$

C

11. A student has 15 video games: 4 adventure games, 4 arcade games, 2 puzzle games, and 5 simulation games. How many ways can the games be positioned on a shelf if the games stay with their genre?

The number of permutations of n different objects is $n!$

Determine the number of ways the games in each genre can be positioned on the shelf:

Adventure games: $4! = 24$

Arcade games: $4! = 24$

Puzzle games: $2! = 2$

Simulation games: $5! = 120$

There are 4 genres. So, the genres can be arranged on the shelf in:

$$4! = 24 \text{ ways}$$

So, the number of ways the games can be positioned on the shelf is:

$$24 \cdot 24 \cdot 24 \cdot 2 \cdot 120 = 3\,317\,760$$

12. a) Seven different keys are to be placed on a key ring. How many ways can the keys be arranged?

Since the key ring is circular, I can always choose the same key to be first, no matter the order of the keys on the key ring. So, I need only consider the arrangements of the remaining 6 keys: $6! = 720$

So, there are 720 ways to arrange 7 keys on a key ring.

- b) How many ways can n distinct keys be arranged on a key ring? Explain.

Similarly, for n keys:

Since the key ring is circular, I can always choose the same key to be first, no matter the order of the keys on the key ring. So, I need only consider the arrangements of the remaining $(n-1)$ keys: $(n-1)!$

So, there are $(n-1)!$ ways to arrange n keys on a key ring.