Lesson 1.1 Exercises, pages 7–12

A

3. Divide.

a) $(2x^2 - x - 6) \div (x - 2)$

$$\begin{array}{r|l} & 2x + 3 \\ \hline x - 2 & 2x^2 - x - 6 \\ & \underline{2x^2 - 4x} \\ & 3x - 6 \\ & \underline{3x - 6} \\ & 0 \end{array}$$

b) $(x^2 - 49) \div (x + 7)$

$$\frac{x^2 - 49}{x + 7} = \frac{(x - 7)\cancel{(x + 7)}}{\cancel{x + 7}}$$
$$= x - 7$$

c) $(2x^2 + 9x - 18) \div (x + 6)$

$$\begin{array}{r|l} & 2x - 3 \\ \hline x + 6 & 2x^2 + 9x - 18 \\ & \underline{2x^2 + 12x} \\ & -3x - 18 \\ & \underline{-3x - 18} \\ & 0 \end{array}$$

d) $(x^2 - 10x + 25) \div (x - 5)$

$$\frac{x^2 - 10x + 25}{x - 5} = \frac{(x - 5)\cancel{(x - 5)}}{\cancel{x - 5}}$$
$$= x - 5$$

4. When each division is performed using synthetic division, the result is as shown. Write the quotient and the remainder.

a) $(2x^3 - 5x^2 + 3x - 7) \div (x - 3)$; 2 1 6 11

The dividend is a polynomial of degree 3, so the quotient is a polynomial of degree 2. The quotient is $2x^2 + x + 6$ and the remainder is 11.

b) $(5x^2 - 7x - 3) \div (x + 4)$; 5 −27 105

The dividend is a polynomial of degree 2, so the quotient is a polynomial of degree 1. The quotient is $5x - 27$ and the remainder is 105.

c) $(3x^4 + 9x^3 - 8x^2 + x - 9) \div (x - 2)$; 3 15 22 45 81

The dividend is a polynomial of degree 4, so the quotient is a polynomial of degree 3. The quotient is $3x^3 + 15x^2 + 22x + 45$ and the remainder is 81.

d) $(-5x^5 - 3x^3 + 11x^2 - 19x) \div (x + 1)$;
−5 5 −8 19 −38 38

The dividend is a polynomial of degree 5, so the quotient is a polynomial of degree 4. The quotient is $-5x^4 + 5x^3 - 8x^2 + 19x - 38$ and the remainder is 38.

5. Divide. Verify your answers.

a) $(x^2 + x - 2) \div (x - 3)$

Compare $x - 3$ to $x - a$: $a = 3$

3	1	1	−2
		3	12
	1	4	10

The quotient is $x + 4$ and the remainder is 10.

Verify:
$(x + 4)(x - 3) + 10$
$= x^2 + x - 12 + 10$
$= x^2 + x - 2$

Since this is the dividend, the answer is correct.

b) $(2x^3 + 5x^2 - 2x + 4) \div (x + 3)$

Compare $x + 3$ to $x - a$: $a = -3$

−3	2	5	−2	4
		−6	3	−3
	2	−1	1	1

The quotient is $2x^2 - x + 1$ and the remainder is 1.

Verify:
$(x + 3)(2x^2 - x + 1) + 1$
$= 2x^3 - x^2 + x + 6x^2 - 3x + 3 + 1$
$= 2x^3 + 5x^2 - 2x + 4$

Since this is the dividend, the answer is correct.

6. A polynomial is divided by $x - 3$. The quotient is $x^2 + 5x - 2$ and the remainder is -3. What is the original polynomial?

Multiply the quotient by the divisor, then add the remainder.

$(x - 3)(x^2 + 5x - 2) + (-3)$

$= x^3 + 5x^2 - 2x - 3x^2 - 15x + 6 - 3$

$= x^3 + 2x^2 - 17x + 3$

The original polynomial is: $x^3 + 2x^2 - 17x + 3$

B

7. Use long division to divide. Write the division statement.

a) $(-8x^2 - 27x + 4x^3 + 45) \div (x - 3)$

Write the polynomial in descending order:

$4x^3 - 8x^2 - 27x + 45$

$$
\begin{array}{r}
4x^2 + 4x - 15 \\
x - 3 \overline{)\, 4x^3 - 8x^2 - 27x + 45} \\
\underline{4x^3 - 12x^2} \qquad\qquad\quad \\
4x^2 - 27x \qquad\quad \\
\underline{4x^2 - 12x} \qquad\quad \\
-15x + 45 \\
\underline{-15x + 45} \\
0
\end{array}
$$

$4x^3 - 8x^2 - 27x + 45 = (x - 3)(4x^2 + 4x - 15)$

b) $(-7x + 2x^4 + 13x^3) \div (x + 2)$

Write the polynomial in descending order:

$2x^4 + 13x^3 - 7x$

Use zeros as placeholders.

$$
\begin{array}{r}
2x^3 + 9x^2 - 18x + 29 \\
x + 2 \overline{)\, 2x^4 + 13x^3 + 0x^2 - 7x + 0} \\
\underline{2x^4 + 4x^3} \qquad\qquad\qquad\qquad \\
9x^3 + 0x^2 \qquad\qquad\quad \\
\underline{9x^3 + 18x^2} \qquad\qquad\quad \\
-18x^2 - 7x \qquad \\
\underline{-18x^2 - 36x} \qquad \\
29x + 0 \\
\underline{29x + 58} \\
-58
\end{array}
$$

$2x^4 + 13x^3 - 7x = (x + 2)(2x^3 + 9x^2 - 18x + 29) - 58$

8. Use synthetic division to divide. Write the division statement.

a) $(-21x^2 + 5x^4 - 12 - 40x + 12x^3) \div (x + 3)$

Write the polynomial in descending order:
$5x^4 + 12x^3 - 21x^2 - 40x - 12$

−3	5	12	−21	−40	−12
		−15	9	36	12
	5	−3	−12	−4	0

$5x^4 + 12x^3 - 21x^2 - 40x - 12 = (x + 3)(5x^3 - 3x^2 - 12x - 4)$

b) $(-11x^3 + 6x^4 + 5 - x^5) \div (1 + x)$

Write the polynomial and binomial in descending order:
$(-x^5 + 6x^4 - 11x^3 + 5) \div (x + 1)$
Use zeros as placeholders.

−1	−1	6	−11	0	0	5
		1	−7	18	−18	18
	−1	7	−18	18	−18	23

$-x^5 + 6x^4 - 11x^3 + 5 = (x + 1)(-x^4 + 7x^3 - 18x^2 + 18x - 18) + 23$

9. Divide the polynomial $2x^5 - x^4 + 2x^3 - 3x^2 + 2x + 10$ by each binomial.

a) $x - 2$

2	2	−1	2	−3	2	10
		4	6	16	26	56
	2	3	8	13	28	66

Result: $2x^4 + 3x^3 + 8x^2 + 13x + 28$ **R66**

b) $4 + x$

Write the binomial in descending order: $x + 4$

−4	2	−1	2	−3	2	10
		−8	36	−152	620	−2488
	2	−9	38	−155	622	−2478

Result: $2x^4 - 9x^3 + 38x^2 - 155x + 622$ **R(−2478)**

10. Look at the exercises for which there was no remainder when a polynomial was divided by a binomial. What relationship is there among the constant terms of the dividend, divisor, and quotient?

The product of the constant terms of the divisor and the quotient is equal to the constant term of the dividend. For example, in question 7a, the constant terms of the divisor and quotient are -3 and -15, and $(-3)(-15) = 45$, which is the constant term of the dividend.

11. **a)** Determine the quotient and remainder when $4x^3 + 5x^2 - 6x + 5$ is divided by each binomial.

i) $x + 1$

-1	4	5	-6	5
		-4	-1	7
	4	1	-7	12

Result: $4x^2 + x - 7$ R12

ii) $x - 1$

1	4	5	-6	5
		4	9	3
	4	9	3	8

Result: $4x^2 + 9x + 3$ R8

b) Use your answers to part a to determine the quotient and remainder when $4x^3 + 5x^2 - 6x + 5$ is divided by each binomial. Explain your strategy.

i) $-x + 1$

$-x + 1 = -(x - 1)$
When the polynomial is divided by $x - 1$, the quotient and remainder are $4x^2 + 9x + 3$ R8. So, when the polynomial is divided by $-x + 1$, the quotient and remainder will be: $-(4x^2 + 9x + 3)$ R8, or $-4x^2 - 9x - 3$ R8

ii) $-x - 1$

$-x - 1 = -(x + 1)$
When the polynomial is divided by $x + 1$, the quotient and remainder are $4x^2 + x - 7$ R12. So, when the polynomial is divided by $-x - 1$, the quotient and remainder will be: $-(4x^2 + x - 7)$ R12, or $-4x^2 - x + 7$ R12

12. Here is a student's solution for dividing $3x^4 - 4x^3 + 5x^2 - 6x + 10$ by $x - 1$ using synthetic division. Identify the error in the solution. Write a correct solution.

$$\begin{array}{r|rrrrr} -1 & 3 & -4 & 5 & -6 & 10 \\ & & -3 & 7 & -12 & 18 \\ \hline & 3 & -7 & 12 & -18 & 28 \end{array}$$

So, $3x^4 - 4x^3 + 5x^2 - 6x + 10 = (x - 1)(3x^3 - 7x^2 + 12x - 18) + 28$

The student should have used 1 instead of -1 because when comparing $x - 1$ to $x - a$, $a = 1$.
Correct solution:

$$\begin{array}{r|rrrrr} 1 & 3 & -4 & 5 & -6 & 10 \\ & & 3 & -1 & 4 & -2 \\ \hline & 3 & -1 & 4 & -2 & 8 \end{array}$$

So, $3x^4 - 4x^3 + 5x^2 - 6x + 10 = (x - 1)(3x^3 - x^2 + 4x - 2) + 8$

13. A polynomial is divided by $x - a$, $a \in \mathbb{Z}$, and has quotient $2x^2 - 3x + 5$ and remainder 7. Determine a possible polynomial. How many different polynomials can you determine? Explain.

Sample response:
The division statement has the form:
Original polynomial = (binomial divisor)(quotient) + remainder
The quotient is: $2x^2 - 3x + 5$
The remainder is: 7
Choose a binomial divisor: $x + 2$
So, the original polynomial is: $(x + 2)(2x^2 - 3x + 5) + 7$
$= 2x^3 - 3x^2 + 5x + 4x^2 - 6x + 10 + 7$
$= 2x^3 + x^2 - x + 17$
I can write many different polynomials because many different binomial divisors are possible.

C

14. Divide.
$(-4x^4 + 17x^2 + 6x^5 - 27x + 10 - 21x^3) \div (x^3 - 2)$

$$\begin{array}{rl} & 6x^2 - 4x - 21 \\ x^3 - 2\,) & \overline{6x^5 - 4x^4 - 21x^3 + 17x^2 - 27x + 10} \\ & \underline{6x^5 \qquad\qquad\qquad - 12x^2} \\ & -4x^4 - 21x^3 + 29x^2 - 27x \\ & \underline{-4x^4 \qquad\qquad\qquad + 8x} \\ & -21x^3 + 29x^2 - 35x + 10 \\ & \underline{-21x^3 \qquad\qquad\qquad + 42} \\ & 29x^2 - 35x - 32 \end{array}$$

The result is: $6x^2 - 4x - 21$ R$(29x^2 - 35x - 32)$