# Lesson 1.2 Exercises, pages 20–25

**A**

**3.** Write each binomial in the form $x - a$. What is the value of $a$?

**a)** $x + 4$

$x + 4 = x - (-4)$
$a = -4$

**b)** $x - 1$

$x - 1$ is in the form $x - a$.
$a = 1$

**c)** $11 + x$

$11 + x = x - (-11)$
$a = -11$

**d)** $-7 + x$

$-7 + x = x - 7$
$a = 7$

**4. a)** Determine the remainder when $x^3 - 4x^2 - 7x + 10$ is divided by each binomial.

**i)** $x - 1$

Let $P(x) = x^3 - 4x^2 - 7x + 10$

$$\begin{aligned} P(1) &= (1)^3 - 4(1)^2 - 7(1) + 10 \\ &= 1 - 4 - 7 + 10 \\ &= 0 \end{aligned}$$

The remainder is 0.

**ii)** $x + 3$

$$\begin{aligned} P(-3) &= (-3)^3 - 4(-3)^2 - 7(-3) + 10 \\ &= -27 - 36 + 21 + 10 \\ &= -32 \end{aligned}$$

The remainder is $-32$.

**iii)** $x + 2$

$$\begin{aligned} &P(-2) \\ &= (-2)^3 - 4(-2)^2 - 7(-2) + 10 \\ &= -8 - 16 + 14 + 10 \\ &= 0 \end{aligned}$$

The remainder is 0.

**iv)** $x - 2$

$$\begin{aligned} &P(2) \\ &= (2)^3 - 4(2)^2 - 7(2) + 10 \\ &= 8 - 16 - 14 + 10 \\ &= -12 \end{aligned}$$

The remainder is $-12$.

**b)** Which binomials in part a are factors of $x^3 - 4x^2 - 7x + 10$? How do you know?

**$x - 1$ and $x + 2$ are factors of $x^3 - 4x^2 - 7x + 10$ because the value of the polynomial when $x = 1$ and when $x = -2$ is 0.**

**5.** Which values of $a$, $a \in \mathbb{Z}$, should be chosen to test for binomial factors of the form $x - a$ of the polynomial $x^4 + 3x^3 - 8x^2 - 12x + 16$? How did you choose the values?

**I chose values of $a$ that are factors of the constant term in the polynomial, 16. Factors of 16 are: 1, −1, 2, −2, 4, −4, 8, −8, 16, −16**

## B

**6. a)** Determine the remainder when each polynomial is divided by $x - 2$.

**i)** $x^2 - 7x + 11$

Let $P(x) = x^2 - 7x + 11$
$P(2) = (2)^2 - 7(2) + 11$
$= 4 - 14 + 11$
$= 1$
The remainder is 1.

**ii)** $2x^3 - 3x^2 - 6x + 8$

Let $P(x) = 2x^3 - 3x^2 - 6x + 8$
$P(2) = 2(2)^3 - 3(2)^2 - 6(2) + 8$
$= 16 - 12 - 12 + 8$
$= 0$
The remainder is 0.

**iii)** $3x^3 - 2x^2 - 10x + 6$

Let $P(x) = 3x^3 - 2x^2 - 10x + 6$
$P(2) = 3(2)^3 - 2(2)^2 - 10(2) + 6$
$= 24 - 8 - 20 + 6$
$= 2$
The remainder is 2.

**iv)** $x^4 - 2x^3 + 3x^2 - 8$

Let $P(x) = x^4 - 2x^3 + 3x^2 - 8$
$P(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 8$
$= 16 - 16 + 12 - 8$
$= 4$
The remainder is 4.

**b)** Explain the relationship between the remainder when a polynomial $P(x)$ is divided by $x - a$, $a \in \mathbb{Z}$, and $P(a)$.

When a polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.
This result comes from the division statement: $P(x) = (x - a)Q(x) + R$
When $x = a$, $x - a = 0$, so $(x - a)Q(x) = 0$
Then, $P(a) = R$

**7.** Determine the remainder.

**a)** $(2x^3 - x^2 + 3x - 2) \div (x - 3)$

Let $P(x) = 2x^3 - x^2 + 3x - 2$
$$\begin{aligned} P(3) &= 2(3)^3 - (3)^2 + 3(3) - 2 \\ &= 54 - 9 + 9 - 2 \\ &= 52 \end{aligned}$$
The remainder is 52.

**b)** $(3x^3 - 2x^2 - 4x + 6) \div (x - 2)$

Let $P(x) = 3x^3 - 2x^2 - 4x + 6$
$$\begin{aligned} P(2) &= 3(2)^3 - 2(2)^2 - 4(2) + 6 \\ &= 24 - 8 - 8 + 6 \\ &= 14 \end{aligned}$$
The remainder is 14.

**8.** When $2x^3 + kx^2 - 3x + 2$ is divided by $x - 2$, the remainder is 4. Determine the value of $k$.

Let $P(x) = 2x^3 + kx^2 - 3x + 2$
$$\begin{aligned} P(2) &= 2(2)^3 + k(2)^2 - 3(2) + 2 \\ &= 16 + 4k - 6 + 2 \\ &= 12 + 4k \end{aligned}$$
The remainder is 4.

So, $12 + 4k = 4$ Solve for $k$.
$$\begin{aligned} 4k &= -8 \\ k &= -2 \end{aligned}$$
The value of $k$ is $-2$.

**9.** Determine one binomial factor of each polynomial.

**a)** $x^4 + 6x^3 + 5x^2 - 24x - 36$

Sample response:
Let $P(x) = x^4 + 6x^3 + 5x^2 - 24x - 36$
The factors of $-36$ are: 1, $-1$, 2, $-2$, 3, $-3$, 4, $-4$, 6, $-6$, 9, $-9$, 12, $-12$, 18, $-18$, 36, $-36$
Use mental math to substitute $x = 1$, then $x = -1$ to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 + 6(2)^3 + 5(2)^2 - 24(2) - 36$
$= 0$
So, $x - 2$ is a factor of $x^4 + 6x^3 + 5x^2 - 24x - 36$.

**b)** $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

Sample response:
Let $P(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$
The factors of 12 are: 1, $-1$, 2, $-2$, 3, $-3$, 4, $-4$, 6, $-6$, 12, $-12$
Use mental math to substitute $x = 1$:
$P(1) = 0$
So, $x - 1$ is a factor of $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$.

**10.** **a)** Show that $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.

**Let $P(x) = x^3 + 4x^2 - 11x - 30$**

$$P(-5) = (-5)^3 + 4(-5)^2 - 11(-5) - 30$$
$$= 0$$

**The remainder is 0, so $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.**

**b)** Determine the other binomial factors of the polynomial. Verify that the factors are correct.

**Divide by $x + 5$ to determine the other factor.**

$$\begin{array}{r|rrrr} -5 & 1 & 4 & -11 & -30 \\ & & -5 & 5 & 30 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

**So, $x^3 + 4x^2 - 11x - 30 = (x + 5)(x^2 - x - 6)$**
**Factor the trinomial.**
$x^2 - x - 6 = (x + 2)(x - 3)$
**So, $x^3 + 4x^2 - 11x - 30 = (x + 2)(x - 3)(x + 5)$**
**To verify, expand:**

$$\begin{aligned}(x + 2)(x - 3)(x + 5) &= (x^2 - x - 6)(x + 5) \\ &= x^3 + 5x^2 - x^2 - 5x - 6x - 30 \\ &= x^3 + 4x^2 - 11x - 30\end{aligned}$$

**Since this is the original polynomial, the factors are correct.**

**11.** Fully factor each polynomial.

**a)** $x^3 + 6x^2 + 3x - 10$

**Let $P(x) = x^3 + 6x^2 + 3x - 10$**
**The factors of $-10$ are: 1, $-1$, 2, $-2$, 5, $-5$, 10, $-10$**
**Use mental math to substitute $x = 1$:**
$P(1) = 0$
**So, $x - 1$ is a factor.**
**Divide to determine the other factor.**

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 3 & -10 \\ & & 1 & 7 & 10 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

**So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x^2 + 7x + 10)$**
**Factor the trinomial: $x^2 + 7x + 10 = (x + 2)(x + 5)$**
**So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x + 2)(x + 5)$**

**b)** $x^4 - 5x^2 + 4$

Let $P(x) = x^4 - 5x^2 + 4$
The factors of 4 are: 1, −1, 2, −2, 4, −4
Use mental math to substitute $x = 1$:
$P(1) = 0$; so, $x - 1$ is a factor.
Use mental math to substitute $x = -1$:
$P(-1) = 0$; so, $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 - 5(2)^2 + 4$
$= 0$
So, $x - 2$ is a factor.
Try $x = -2$: $P(-2) = (-2)^4 - 5(-2)^2 + 4$
$= 0$
So, $x + 2$ is a factor.
Since the original polynomial has degree 4, it can have at most 4 binomial factors.
So, $x^4 - 5x^2 + 4 = (x - 1)(x + 1)(x - 2)(x + 2)$

**12.** **a)** What value of $b$ will ensure $x + 3$ is a factor of $bx^3 - 2x^2 + x - 6$?

Let $P(x) = bx^3 - 2x^2 + x - 6$
If $x + 3$ is a factor, $P(-3) = 0$
$P(-3) = b(-3)^3 - 2(-3)^2 + (-3) - 6$
$= -27b - 27$
Let $P(-3) = 0$
$-27b - 27 = 0$
$b = -1$
So, the value of $b$ is −1.

**b)** What value of $d$ will ensure $x + 2$ is a factor of $3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$?

Let $P(x) = 3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$
If $x + 2$ is a factor, $P(-2) = 0$
$P(-2) = 3(-2)^5 - d(-2)^4 + 4(-2)^3 - 2d(-2)^2 + (-2) + 10$
$= -120 - 24d$
Let $P(-2) = 0$
$-120 - 24d = 0$
$d = \frac{120}{-24}$, or −5
So, the value of $d$ is −5.

**13.** Determine whether $x + b$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

Let $P(x) = (x + b)^5 + (x + p)^5 + (b - p)^5$
If $x + b$ is a factor, $P(-b) = 0$
$P(-b) = (-b + b)^5 + (-b + p)^5 + (b - p)^5$
$= 0 + (-(b - p))^5 + (b - p)^5$
$= 0 - (b - p)^5 + (b - p)^5$
$= 0$
Since the remainder is 0, $(x + b)$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

**C**

**14.** When $mx^3 - 2x^2 + nx - 4$ is divided by $x + 2$, the remainder is 4. When $mx^3 - 2x^2 + nx - 4$ is divided by $x - 1$, the remainder is $-11$. Determine the values of $m$ and $n$.

Let $P(x) = mx^3 - 2x^2 + nx - 4$

$P(-2) = 4$

$P(-2) = m(-2)^3 - 2(-2)^2 + n(-2) - 4$

$4 = -8m - 2n - 12$

$0 = -8m - 2n - 16$ ①

$P(1) = -11$

$P(1) = m(1)^3 - 2(1)^2 + n(1) - 4$

$-11 = m + n - 6$

$0 = m + n + 5$ ②

Solve the system of equations:

$0 = -8m - 2n - 16$ ①

$0 = m + n + 5$ ②

Solve equation ② for $m$: $m = -n - 5$

Substitute for $m$ in equation ①.

$0 = -8m - 2n - 16$ ①

$0 = -8(-n - 5) - 2n - 16$

$0 = 8n + 40 - 2n - 16$

$0 = 6n + 24$

$n = -4$

Substitute $n = -4$ in equation ②.

$0 = m - 4 + 5$

$m = -1$

So, $m = -1$ and $n = -4$

**15.** Determine each remainder.

**a)** $(8x^2 - 6x + 3) \div (4x + 1)$

$$\begin{array}{r}
2x - 2 \\
4x + 1 \overline{)\, 8x^2 - 6x + 3} \\
\underline{8x^2 + 2x} \quad\ \ \\
-8x + 3 \\
\underline{-8x - 2} \\
5
\end{array}$$

The remainder is 5.

**b)** $(3x^3 + 2x^2 - 6x - 1) \div (3x + 2)$

$$\begin{array}{r}
x^2 \qquad\quad - 2 \\
3x + 2 \overline{)\, 3x^3 + 2x^2 - 6x - 1} \\
\underline{3x^3 + 2x^2} \qquad\qquad \\
0 - 6x - 1 \\
\underline{- 6x - 4} \\
3
\end{array}$$

The remainder is 3.