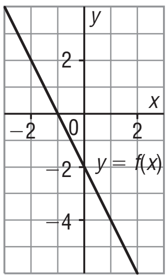
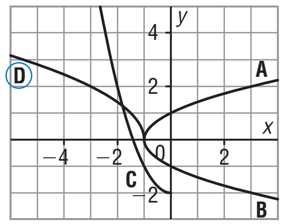
Checkpoint: Assess Your Understanding, pages 124–128

2.1

1. **Multiple Choice** Given the graph of the function $y = f(x)$, which graph below right represents $y = \sqrt{f(x)}$?

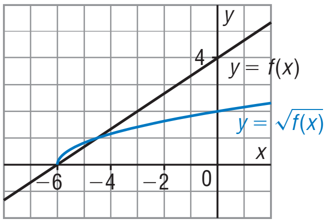




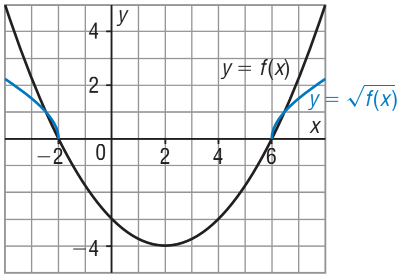
2. For each function $y = f(x)$ graphed below:

- Sketch the graph of $y = \sqrt{f(x)}$.
- State the domain and range of $y = \sqrt{f(x)}$.
- Explain why the domains are different and the ranges are different.

a)



b)



Mark points where $y = 0$ or $y = 1$. The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$ between these points. Choose, then mark other points on the graph of $y = \sqrt{f(x)}$.

x	$y = f(x)$	$y = \sqrt{f(x)}$
-3	2	$\doteq 1.4$
0	4	2

Join the points with a smooth curve.
Domain is: $x \geq -6$
Range is: $y \geq 0$

$y = \sqrt{f(x)}$ is not defined for $-2 < x < 6$. Mark points where $y = 0$ or $y = 1$. Choose, then mark other points on the graph of $y = \sqrt{f(x)}$.

x	$y = f(x)$	$y = \sqrt{f(x)}$
-4	5	$\doteq 2.2$
8	5	$\doteq 2.2$

Join the points with 2 smooth curves.
Domain is: $x \leq -2$ or $x \geq 6$
Range is: $y \geq 0$

The domain of a linear function or a quadratic function is all real values of x, but the square root of a negative number is undefined, so any value of x that makes the radicand negative is not in the domain of a radical function.
The range of the linear function is all real values of y, and the range of the quadratic function is all real values of y that are greater than or equal to -4. The principal square root of a number is always 0 or positive, so the range of the radical functions is restricted to these values of y.

3. Solve each radical equation by graphing. Give the solution to the nearest tenth.

a) $-x + 3 = \sqrt{2x - 1}$

Write the equation as:
$-x + 3 - \sqrt{2x - 1} = 0$
Graph the related function:
$f(x) = -x + 3 - \sqrt{2x - 1}$
Use graphing technology to determine the approximate zero: 1.5505103
So, the solution is: $x \doteq 1.6$

b) $\sqrt{x + 2} = 5 - \sqrt{3x + 4}$

Write the equation as:
$\sqrt{x + 2} - 5 + \sqrt{3x + 4} = 0$
Graph the related function:
$f(x) = \sqrt{x + 2} - 5 + \sqrt{3x + 4}$
Use graphing technology to determine the approximate zero: 1.779514
So, the solution is: $x \doteq 1.8$

2.2

4. Use graphing technology to graph each rational function. Identify any non-permissible values of x and the equations of any horizontal asymptotes.

a) $y = \frac{3x}{x + 4}$

Since $x + 4 \neq 0$, then $x \neq -4$
The vertical asymptote has equation $x = -4$.
The horizontal asymptote has equation $y = 3$.

b) $y = \frac{3x}{x^2 - 4}$

Since $x^2 - 4 \neq 0$, then $x \neq \pm 2$
The vertical asymptotes have equations $x = 2$ and $x = -2$.
The horizontal asymptote has equation $y = 0$.

c) $y = \frac{x^2 - 4}{3x}$

Since $3x \neq 0$, then $x \neq 0$
The vertical asymptote has equation $x = 0$.
There is no horizontal asymptote.

d) $y = \frac{x^2 - 4x}{3x}$

Since $3x \neq 0$, then $x \neq 0$
There is a hole at $x = 0$.
There is no horizontal asymptote.

2.3

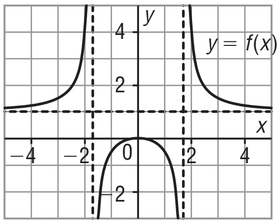
5. Multiple Choice Which function has a graph with a hole?

A. $y = \frac{x + 4}{2x^2 + 8x}$ **B.** $y = \frac{x - 4}{2x^2 + 8x}$

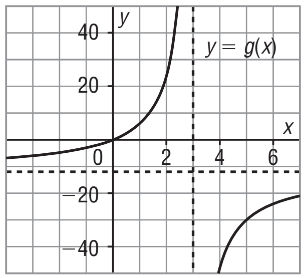
C. $y = \frac{4x + 4}{2x^2 + 8x}$ **D.** $y = \frac{x + 4}{2x^2 - 8x}$

6. Match each function to its graph. Justify your choice.

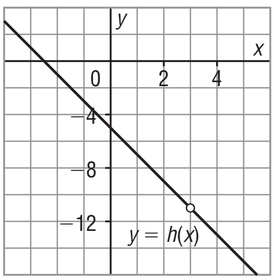
i) Graph A



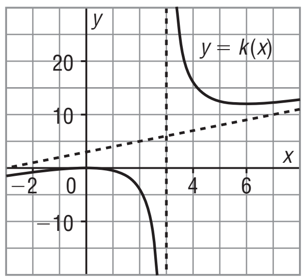
ii) Graph B



iii) Graph C



iv) Graph D



a) $y = \dfrac{-12x}{x - 3}$

There is a vertical asymptote with equation $x = 3$. The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the x-axis. The function matches Graph B.

b) $y = \dfrac{2x^2 - x - 15}{3 - x}$

Factor: $y = \dfrac{(2x + 5)(x - 3)}{3 - x}$,

or $y = \dfrac{-(2x + 5)(x - 3)}{x - 3}$

There is a hole at $x = 3$. The function matches Graph C.

c) $y = \dfrac{x^2}{x - 3}$

There is a vertical asymptote with equation $x = 3$.
The degree of the numerator is 1 more than the degree of the denominator, so there is an oblique asymptote.
The function matches Graph D.

d) $y = \dfrac{x^2}{x^2 - 3}$

The function is not defined for $x^2 - 3 = 0$; that is, $x = \pm\sqrt{3}$. So, there are vertical asymptotes at $x = -\sqrt{3}$ and $x = \sqrt{3}$. The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the x-axis. The function matches Graph A.

7. For the graph of each rational function below, determine without technology:

i) the equations of any asymptotes and the coordinates of any hole

ii) the domain of the function

Use graphing technology to verify the characteristics.

a) $y = \frac{2x^2}{25 - x^2}$

i) The function is undefined when $25 - x^2 = 0$; that is, when $x = \pm 5$. There are no common factors, so there are vertical asymptotes with equations $x = 5$ and $x = -5$.
The degrees of the numerator and denominator are equal, so there is a horizontal asymptote. The leading coefficients of the numerator and denominator are 2 and -1, respectively.
So, the horizontal asymptote has equation: $y = -2$

ii) The domain is: $x \neq \pm 5$

b) $y = \frac{-2x^2 - 6x}{x + 3}$

i) The function is undefined when $x + 3 = 0$; that is, when $x = -3$.
Factor: $y = \frac{-2x(x + 3)}{x + 3}$
There is a hole at $x = -3$. The function is: $y = -2x$, $x \neq -3$
The coordinates of the hole are: $(-3, 6)$

ii) The domain is: $x \neq -3$

8. Solve each rational equation by graphing. Give the solution to the nearest tenth.

a) $x - 2 = \frac{3x - 5}{x - 3}$

Graph a related function:

$f(x) = x - 2 - \frac{3x - 5}{x - 3}$

Use graphing technology to determine the zeros:
$x \doteq 1.8$ or $x \doteq 6.2$

b) $\frac{x^2 + 3x - 5}{x - 1} = -5$

Graph a related function:

$f(x) = \frac{x^2 + 3x - 5}{x - 1} + 5$

Use graphing technology to determine the zeros:
$x \doteq -9.1$ or $x \doteq 1.1$