Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2015

Manitoba

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Disponible en français.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Advanced Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

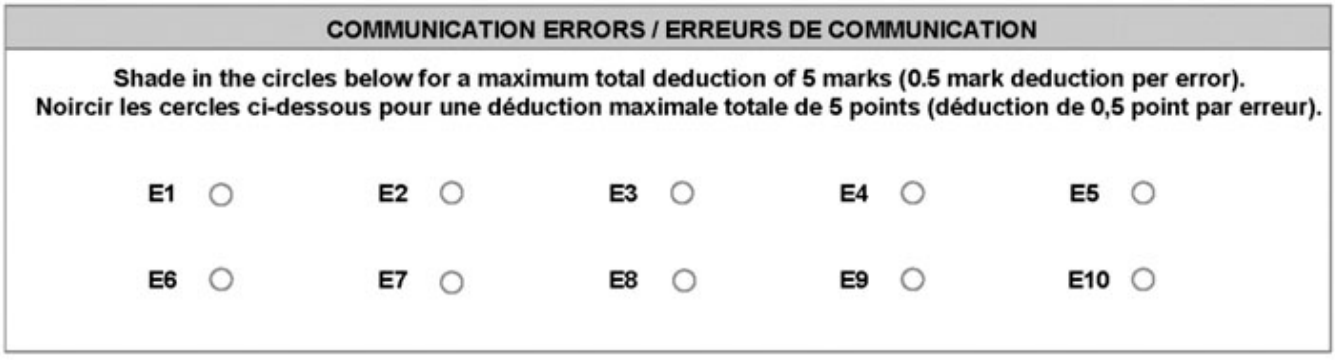
The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.



Example: Marks assigned to the student.

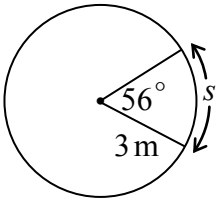
	Booklet 1	Selected Response	Booklet 2	Communication Errors (Deduct)	Total
Marks Awarded	25	7	40	1½	70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines

Booklet 1 Questions

Question 1 T1

Use the information in the diagram to determine the value of the arc length "s", given the central angle of 56°.



Solution

$\theta = 56^\circ \times \dfrac{\pi}{180^\circ}$ 1 mark for conversion

$\theta = \dfrac{56\pi}{180}$ or $\dfrac{14\pi}{45}$

$s = \theta r$

$s = \left(\dfrac{14\pi}{45}\right)(3\,\text{m})$ 1 mark for substitution

$s = \dfrac{14\pi}{15}\,\text{m}$ or $2.932\,\text{m}$ **2 marks**

Exemplar 1

$S = \theta r$

$= (56^\circ)(3)$

$\boxed{S = 168\text{m}}$

1 out of 2

+ 1 mark for substitution

Exemplar 2

$S = \theta r$

$56^\circ\left(\pi/180\right) = 56\pi/180 = 28\pi/90$

$14\pi/45$

$S = \left(\frac{14\pi}{45}\right)(3\text{m})$

$S = \frac{42\pi}{45}$

2 out of 2

award full marks
E5 (missing unit of measure)

Solve $\tan^2\theta - 5\tan\theta + 4 = 0$ where $\theta \in \mathbb{R}$.

Solution

Method 1

$(\tan\theta - 1)(\tan\theta - 4) = 0$

$\tan\theta = 1$ $\quad$ $\tan\theta = 4$

$\theta_r = 1.3258$

1 mark for solving for $\tan\theta$ (½ mark for each branch)

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad$ $\theta = 1.326,\ 4.467$

$\theta = 0.785,\ 3.927$

2 marks (½ mark for each value of θ)

$$\left.\begin{aligned}\theta &= \frac{\pi}{4} + 2k\pi \\ &= 1.326 + 2k\pi \\ &= \frac{5\pi}{4} + 2k\pi \\ &= 4.467 + 2k\pi\end{aligned}\right\} \text{ where } k \in \mathrm{I}$$

1 mark for general solution

4 marks

Method 2

$(\tan\theta - 1)(\tan\theta - 4) = 0$

$\tan\theta = 1$ $\quad$ $\tan\theta = 4$

$\theta_r = 1.3258$

1 mark for solving for $\tan\theta$ (½ mark for each branch)

$\theta = \frac{\pi}{4}$ $\quad$ $\theta = 1.326$

$\theta = 0.785$

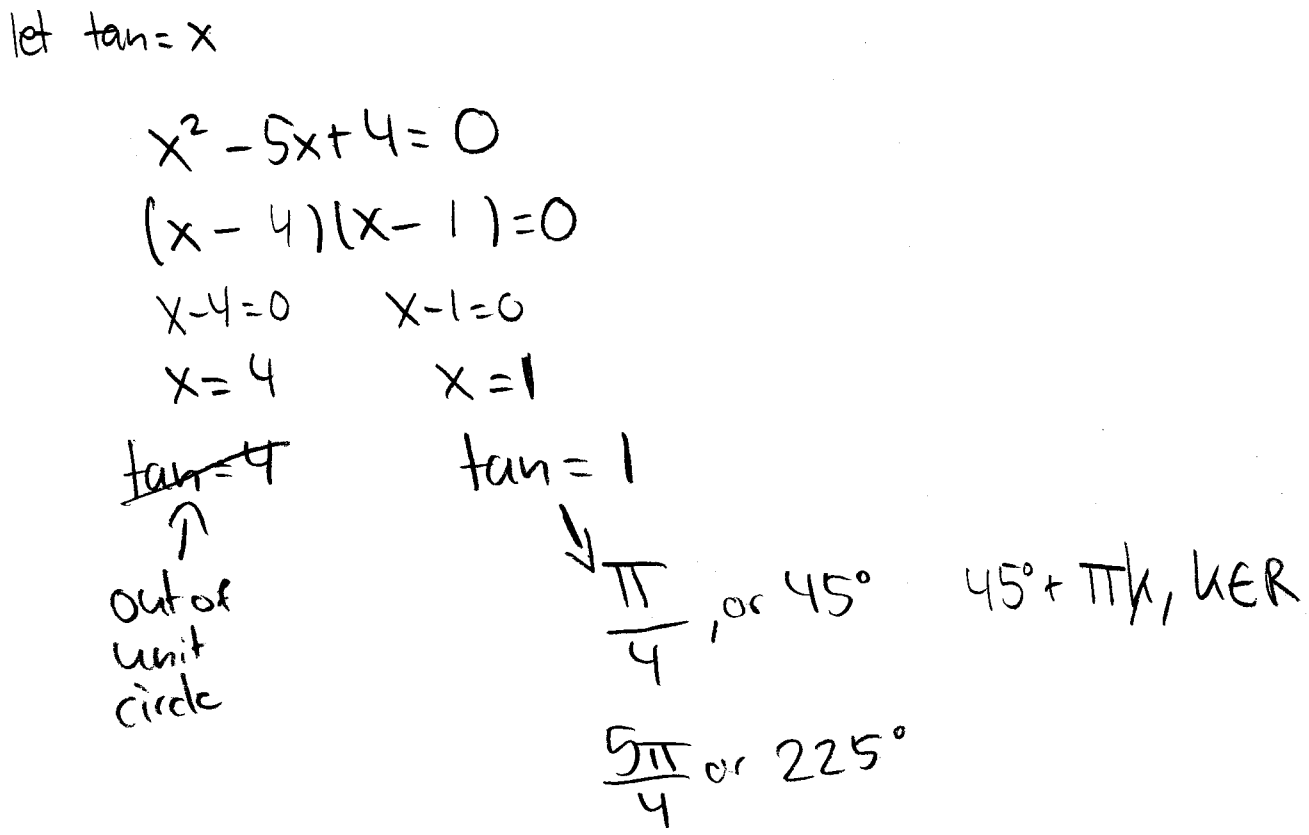
2 marks (1 mark for each value of θ)

$$\left.\begin{aligned}\theta &= \frac{\pi}{4} + k\pi \\ &= 1.326 + k\pi\end{aligned}\right\} \text{ where } k \in \mathrm{I}$$

1 mark for general solution

4 marks

Exemplar 1



2 out of 4

+ 1 mark for solving for $\tan\theta$
+ 1 mark for values of θ
E3 (variable omitted in lines 1 and 6)
E2 (changing an equation to an expression in line 7)

Exemplar 2

$\tan^2\theta - 5\tan\theta + 4 = 0$

$(\tan\theta - 1)(\tan\theta - 5) = 0$

$\tan\theta = 1 \qquad \tan\theta = 5$

$\theta_R = 0.785 \qquad \theta_R = 1.3734$

$\theta_I = 0.7853 \qquad \theta_I = 1.3734$

$\theta_{III} = 0.7853 + \pi \qquad \theta_{III} = 1.3734 + \pi$

$\theta_{III} = 3.9269 \qquad \theta_{III} = 4.5150$

2½ out of 4

+ 1 mark for solving for $\tan\theta$
+ 2 marks for consistent values of θ
– ½ mark for arithmetic error in line 2

Exemplar 3

$\tan\theta = x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4 \qquad x = 1$

$\tan\theta = 4 \qquad \tan\theta = 1$

$\theta = 75.964° \qquad \theta = 45°$

2 out of 4

+ 1 mark for solving for $\tan\theta$
+ 1 mark for values of θ

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Solve:

$$2^{5x} = 3(5)^{x-3}$$

Solution

$$\log 2^{5x} = \log\left[3(5)^{x-3}\right]$$

½ mark for applying logarithms

$$5x\log 2 = \log 3 + (x-3)\log 5$$

1 mark for product rule
1 mark for power rule

$$5x\log 2 = \log 3 + x\log 5 - 3\log 5$$

$$5x\log 2 - x\log 5 = \log 3 - 3\log 5$$

½ mark for collecting like terms

$$x(5\log 2 - \log 5) = \log 3 - 3\log 5$$

$$x = \frac{\log 3 - 3\log 5}{5\log 2 - \log 5}$$

½ mark for isolating x

$$x = -2.009$$

½ mark for evaluating a quotient of logarithms

4 marks

Exemplar 1

$$\log(2^{5x}) = \log(3(5^{x-3}))$$
$$5x\log 2 = x-3\log(3\cdot 5)$$
$$5x\log 2 = x-3\log(15)$$

1½ out of 4

+ ½ mark for applying logarithms
+ 1 mark for power rule
E7 (notation error in line 2)

Exemplar 2

$$(5x)\log 2 = \log 3 + (x-3)\log 5$$
$$(5x)\log 2 - (x-3)\log 5 = \log 3$$
$$(5x) - (x-3) = \frac{\log 3}{\log 2 - \log 5}$$
$$4x+3 = -1,1990$$
$$4x = 1,8010$$
$$x = 0,4503$$

2½ out of 4

+ ½ mark for applying logarithms
+ 1 mark for product rule
+ 1 mark for power rule
+ ½ mark for evaluating a quotient of logarithms
– ½ mark for arithmetic error in line 5

Exemplar 3

$$2^{5x} = 15^{x-3}$$

$$\log 2^{5x} = \log 15^{x-3}$$

$$5x \log 2 = (x-3) \log 15$$

$$5x \log 2 = x \log 15 - 3 \log 15$$

$$5x \log 2 - x \log 15 = -3 \log 15$$

$$x \frac{(5 \log 2 - \log 15)}{5 \log 2 - \log 15} = \frac{-3 \log 15}{5 \log 2 - \log 15}$$

$$x = -10,722$$

3 out of 4

+ ½ mark for applying logarithms
+ 1 mark for power rule
+ ½ mark for collecting like terms
+ ½ mark for isolating x
+ ½ mark for evaluating a quotient of logarithms

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David and Sarah are in a class of 10 boys and 8 girls.
A committee of 3 boys and 2 girls is to be selected from the students in this class.

Determine the number of possible committees if David and Sarah cannot be on the same committee.

Solution

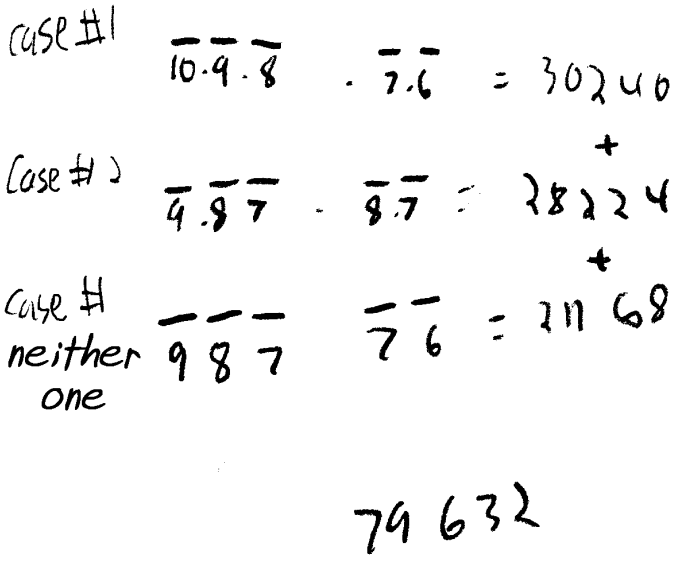
Method 1

All:	${}_{10}C_3 \times {}_8C_2 = 3360$	1 mark for all possible committees
Both:	${}_9C_2 \times {}_7C_1 = 252$	1 mark for both on the committee
	$3360 - 252 = 3108$	1 mark for subtraction of cases
		3 marks

Method 2

Case 1: David, not Sarah	${}_9C_2 \times {}_7C_2 = 756$	½ mark for Case 1
Case 2: Sarah, not David	${}_9C_3 \times {}_7C_1 = 588$	½ mark for Case 2
Case 3: Neither David nor Sarah	${}_9C_3 \times {}_7C_2 = 1764$	1 mark for Case 3
	$756 + 588 + 1764 = 3108$	1 mark for addition of cases
		3 marks

Exemplar 1

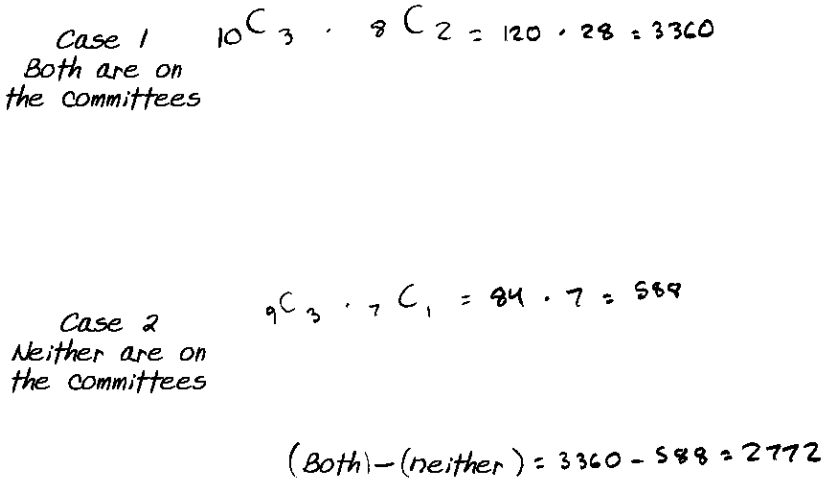


1 out of 3

Method 2

+ 1 mark for Case 3
+ 1 mark for addition of cases
− 1 mark for concept error (permutations instead of combinations)

Exemplar 2



2 out of 3

+ 1 mark for all possible committees
+ 1 mark for subtraction of cases

Exemplar 3

P(David only) + P(Sarah only) + P(no David, no Sarah)

$= ({}_9C_2 \cdot {}_8C_2) + ({}_{10}C_3 \cdot {}_7C_1) + ({}_9C_3 \cdot {}_7C_2)$

$= (36 \cdot 28) + (120 \cdot 7) + (84 \cdot 21)$

$= 1008 + 840 + 1764$

$= 3612$ committee possibilities

2 out of 3

+ 1 mark for Case 3
+ 1 mark for addition of cases

Exemplar 4

David: $({}_{10}C_3)({}_7C_2) = 2520$

Sarah: $({}_9C_3)({}_8C_2) = 2352$

$2520 + 2352 = \boxed{4872}$

1 out of 3

+ 1 mark for addition of cases

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In the binomial expansion of $\left(\frac{3}{x^2} - x^5\right)^{10}$, simplify the 7th term.

Solution

$$t_7 = {}_{10}C_6\left(\frac{3}{x^2}\right)^4\left(-x^5\right)^6$$

$$t_7 = 210\left(\frac{81}{x^8}\right)\left(x^{30}\right)$$

$$t_7 = 17\ 010x^{22}$$

2 marks (1 mark for ${}_{10}C_6$, ½ mark for each consistent factor)

1 mark for simplification (½ mark for coefficient, ½ mark for exponent)

3 marks

Exemplar 1

$$t_{k+1} = {}_nC_k\, a^{n-k} b^k$$

$$t_{7+1} = {}_{10}C_7 \left(\frac{3}{x^2}\right)^{10-7} (x^5)^7$$

$$= 120\left(\frac{3}{x^2}\right)^3 (x^5)^7$$

$$= 120\left(\frac{9}{x^5}\right)(x^{12})$$

$$= \frac{1080}{x^5}(x^{12})$$

$$= \frac{1080x^{12}}{x^5}$$

$$= 1080x^{12-5}$$

$$= 1080x^7$$

½ out of 3

+ ½ mark (½ mark for each consistent factor)

Exemplar 2

$t_{6+1} = {}_{10}C_6 \cdot \frac{3}{x^2}^{10-6} \cdot (x^5)^6$

$t_7 = 210 \cdot \frac{3}{x^2}^4 \cdot -x^{11}$

$t_7 = \frac{210}{1} \cdot \frac{81}{x^6} \cdot -x^{11}$

$t_7 = \frac{17010}{x^6} \cdot -x^{11}$

$t_7 = \boxed{\frac{17010}{-x^{17}}}$

2 out of 3

+ 2 marks (1 mark for ${}_{10}C_6$, ½ mark for each consistent factor)

E4 (missing brackets but still implied in line 1)

Exemplar 3

$t_{k+1} = {}_nC_k \, a^{n-k} \, b^k$

$t_{6+1} = {}_{10}C_6 \, a^{10-6} \, b^6$

$= {}_{10}C_6 \left(\frac{3}{x^2}\right)^4 (x^5)^6$

$= 210\left(\frac{81}{x^6}\right)(x^{11})$

$= 210(81x^5)$

$= 17010x^5$

2 out of 3

+ 1 mark for ${}_{10}C_6$

+ ½ mark for $\left(\frac{3}{x^2}\right)^4$

+ ½ mark for simplification of coefficient

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A lake affected by acid rain has a pH of 4.4.
A person suffering from heartburn has a stomach acid pH of 1.2.
The pH of a solution is defined as $\text{pH} = -\log\left[\text{H}^+\right]$ where $\left[\text{H}^+\right]$ is the hydrogen ion concentration.

How many times greater is the hydrogen ion concentration of the stomach than that of the lake?

Express your answer as a whole number.

Solution

Method 1

$$\text{pH} = -\log\left[\text{H}^+\right]$$
$$-\text{pH} = \log\left[\text{H}^+\right]$$
$$\left[\text{H}^+\right] = 10^{-\text{pH}}$$

1 mark for exponential form

$$\frac{\left[\text{H}^+\right]_{\text{stomach}}}{\left[\text{H}^+\right]_{\text{lake}}} = \frac{10^{-1.2}}{10^{-4.4}}$$

1 mark for comparison

$$= 10^{3.2}$$
$$= 1584.9$$
$$= 1585$$

2 marks

Method 2

Lake

$$4.4 = -\log\left[\text{H}^+\right]$$
$$-4.4 = \log\left[\text{H}^+\right]$$
$$10^{-4.4} = \left[\text{H}^+\right]$$

½ mark for exponential form

Stomach

$$1.2 = -\log\left[\text{H}^+\right]$$
$$-1.2 = \log\left[\text{H}^+\right]$$
$$10^{-1.2} = \left[\text{H}^+\right]$$

½ mark for exponential form

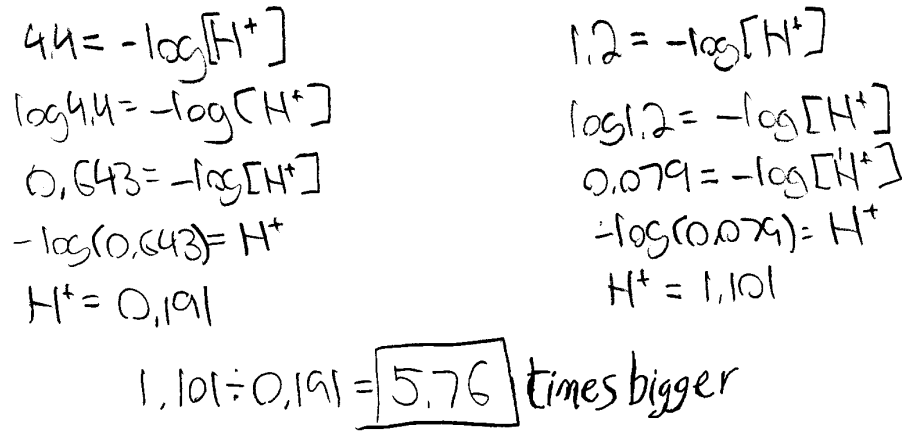
$$\frac{\left[\text{H}^+\right]_{\text{stomach}}}{\left[\text{H}^+\right]_{\text{lake}}} = \frac{10^{-1.2}}{10^{-4.4}}$$

1 mark for comparison

$$= 10^{3.2}$$
$$= 1585$$

2 marks

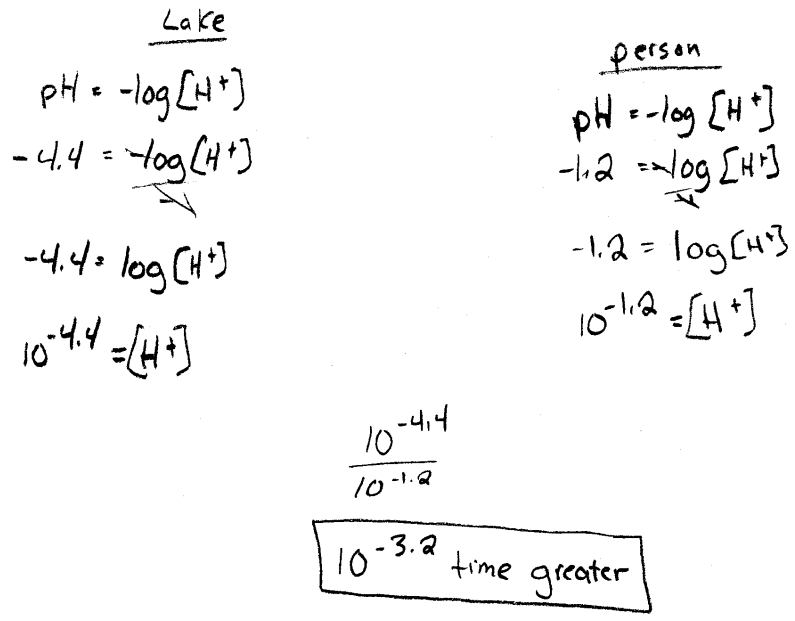
Exemplar 1



1 out of 2

+ 1 mark for comparison
E6 (rounding too early in line 3)

Exemplar 2



1 out of 2

+ 1 mark for exponential form

Exemplar 3

acid rain 4.4

$pH = -\log(H^+)$
$4.4 = -\log(H^+)$
$10^{4.4} = H^+$
$25118.864 = H^+$

stomach acid 1.2

$pH = -\log(H^+)$
$1.2 = -\log(H^+)$
$10^{1.2} = H^+$
$15.849 = H^+$

The hydrogen ion concentration is 25103.015 more concentrated.

½ out of 2

+ 1 mark for exponential form
– ½ mark for arithmetic error in lines 4 and 9

Exemplar 4

$10^{3.2}$

$= 1585$

2 out of 2

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Solve the following equation algebraically over the interval $[0,\ 2\pi]$.

$$\cos 2\theta - 3\sin\theta - 2 = 0$$

Solution

$\left(1 - 2\sin^2\theta\right) - 3\sin\theta - 2 = 0$ — 1 mark for correct substitution of an appropriate identity

$-2\sin^2\theta - 3\sin\theta - 1 = 0$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\qquad$ $\sin\theta = -1$ — 1 mark for solving for $\sin\theta$ (½ mark for each branch)

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\qquad$ $\theta = \frac{3\pi}{2}$ — 2 marks for solutions (1 mark for each branch; ½ mark for each value in the left branch)

4 marks

Exemplar 1

$\cos 2\theta = 1 - 2\sin^2\theta$

$-2\sin^2\theta - 3\sin\theta - 2 + 1$

$2\sin^2\theta - 3\sin\theta + 1$

$(\sin\theta - 2)(\sin\theta - 1)$

~~$\sin\theta = 2$~~ $\sin\theta = 1$

$\sin\theta = \frac{\pi}{2}$

3 out of 4

+ 1 mark for correct substitution of an appropriate identity
+ 1 mark for solving for $\sin\theta$
+ 2 marks for solutions
– ½ mark for arithmetic error in line 3
– ½ mark for arithmetic error in line 4
E2 (changing an equation to an expression in line 2)
E7 (notation error in line 6)

Exemplar 2

$\cos 2\theta - 3\sin\theta - 2 = 0$

$1 - 2\sin^2\theta - 3\sin\theta - 2 = 0$

$-2\sin^2\theta - 3\sin\theta + 1 = 0$

$(-2\sin\theta - 1)(\sin\theta + 1)$

$-2\sin\theta - 1 = 0$ $\quad\quad$ $\sin\theta + 1 = 0$

$\sin\theta = 1$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{6}$ $\quad\quad$ $\theta = \frac{\pi}{2}$

2½ out of 4

+ 1 mark for correct substitution of an appropriate identity
+ ½ mark for solving for $\sin\theta$
+ 1 mark for solution in right branch
E2 (changing an equation to an expression in line 4)

Explain how the value of n affects the behaviour of the graph of the polynomial function $p(x) = (x+3)(x-1)^n$, as $p(x)$ approaches the x-intercept at $x = 1$.

Solution

If n is even, the graph will turn at the x-axis at $x = 1$.
If n is odd, the graph will cross the x-axis at $x = 1$.

1 mark

or

As n increases, $p(x)$ will become flatter around the intercept at $x = 1$.

Exemplar 1

The value "n" will change the multiplicity of the function, therefore depending on what the "n" value is the graph will look different.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

If n is odd, the graph will cross the x-axis at $x = 1$.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

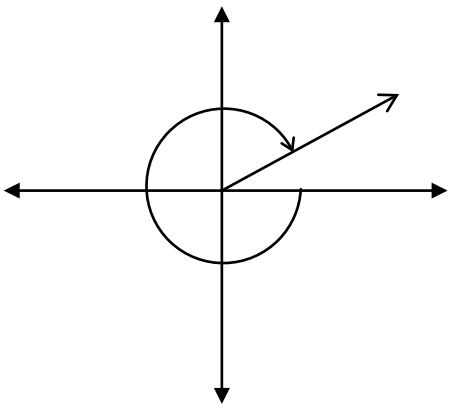
Exemplar 3

* if (n) is an odd number that means the $(x=1)$ will just touch the intercepts.
* And if (n) is an even number that means $(x=1)$ will cross the intercepts.

0 out of 1

Sketch the angle -320° in standard position.

Solution



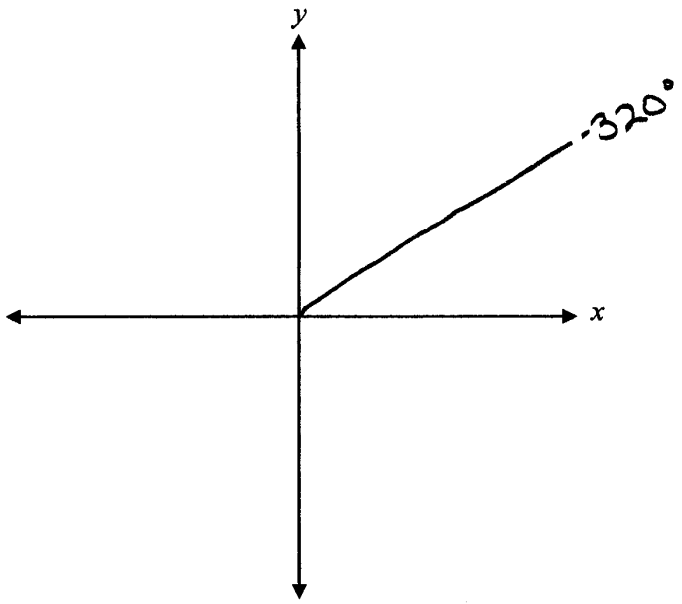
1 mark for angle drawn in quadrant I

1 mark

Note(s):

- If directional arrow is not indicated, it is a communication error, E1 (final answer not stated).

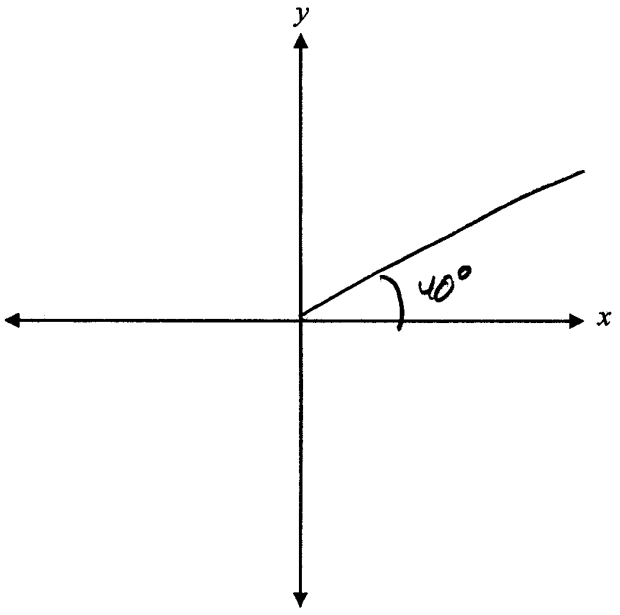
Exemplar 1



1 out of 1

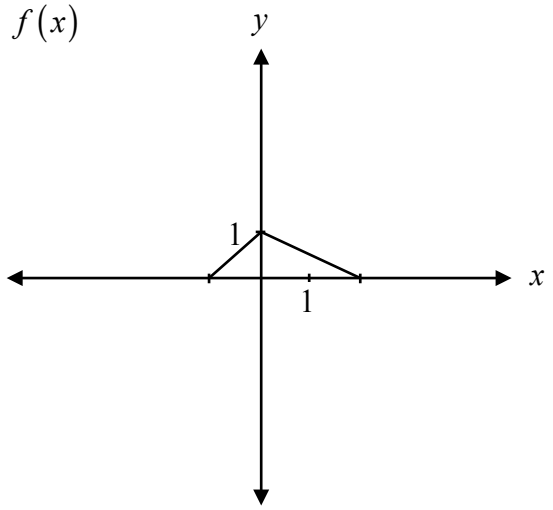
award full marks
E1 (final answer not stated)

Exemplar 2



0 out of 1

Given the graph of $y = f(x)$, explain how to graph $y = f(-x)$.



Solution

Multiply the x-coordinates by -1. **1 mark**

or

Reflect the graph over the y-axis.

Exemplar 1

Since the "x" is now negative, it would be a flip over the x-axis so, you would flip below the x-axis line, so any positive numbers become negative

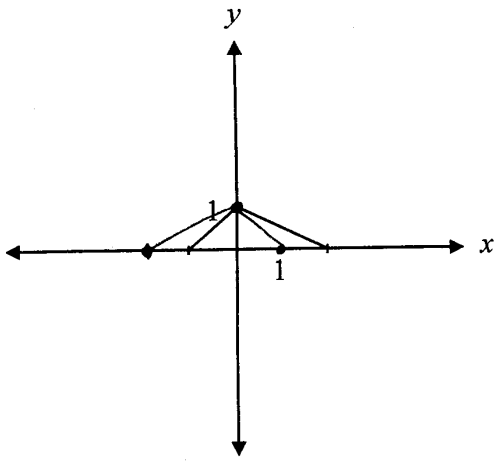
0 out of 1

Exemplar 2

You have to divide the x values by -1.

1 out of 1

Exemplar 3



0 out of 1

Explain how the graph of $y = \frac{3(x-1)}{(x-1)}$ is different than the graph of $y = 3$.

Solution

There is a point of discontinuity (hole) when $x = 1$ on the graph of $y = \frac{3(x-1)}{(x-1)}$. **1 mark**

Exemplar 1

The graph of $\frac{3(x-1)}{(x-1)}$ will have non permissable values, as you can not divide by zero, while the graph of $y=3$ does not have any non permissable values.

1 out of 1

Exemplar 2

It is not different.
$y=3$ is the same as $y=\frac{3(x-1)}{(x-1)}$ because the $(x-1)$ cancels itself.

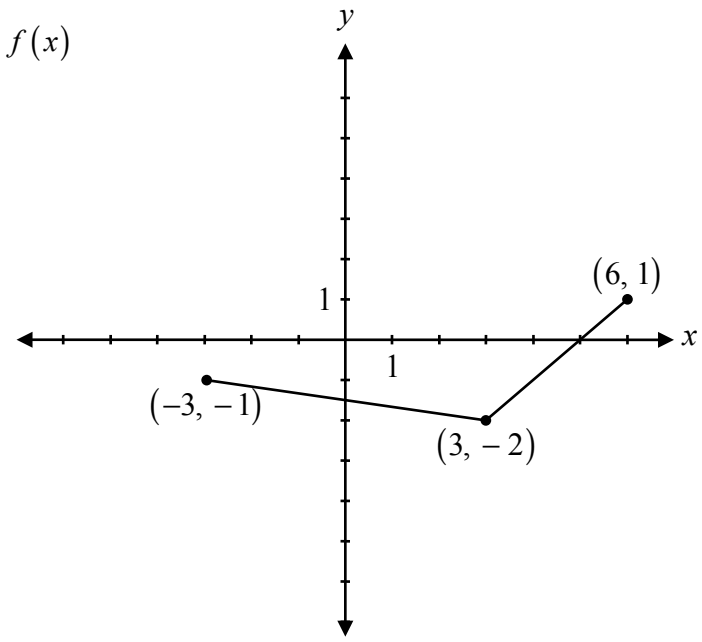
$y=3$ $\quad y=\frac{3(x-1)}{(x-1)}$

$y=\frac{3}{1}$ $\quad y=3$

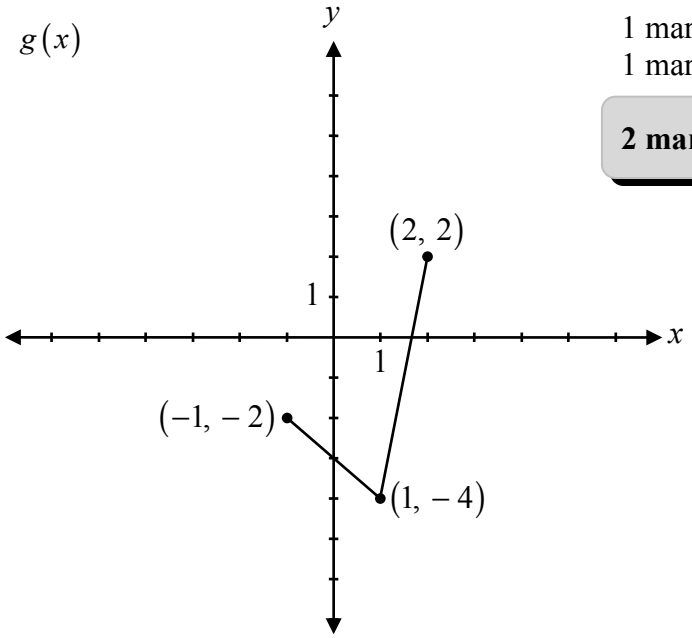
0 out of 1

Question 12 R3

Given the graph of $f(x)$, sketch the graph of $g(x) = 2f(3x)$.



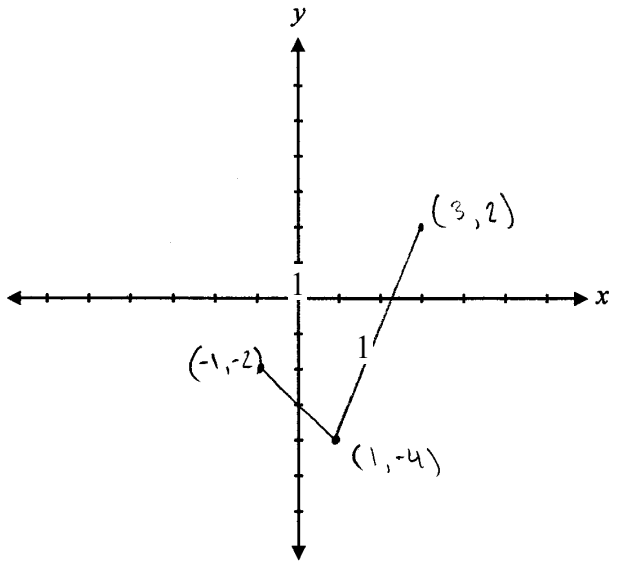
Solution



1 mark for horizontal stretch/compression
1 mark for vertical stretch

2 marks

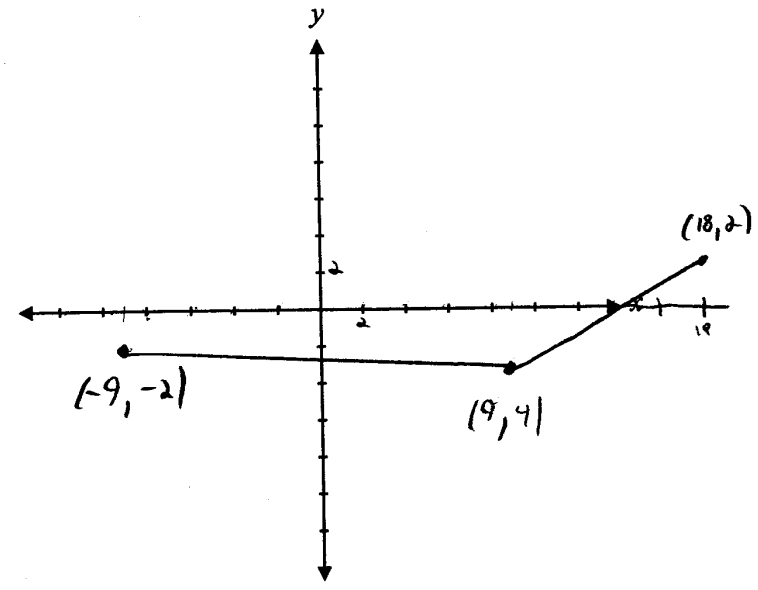
Exemplar 1



1½ out of 2

award full marks
– ½ mark for arithmetic error (one incorrect point)

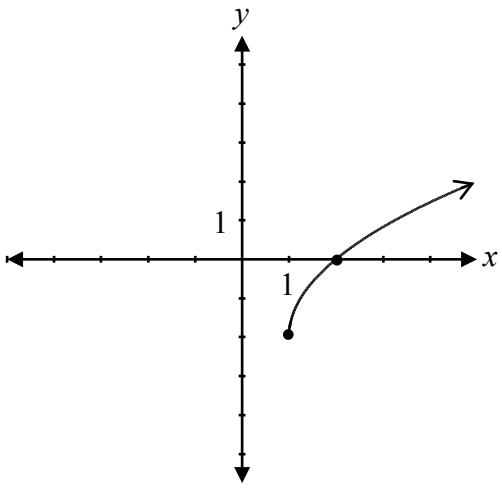
Exemplar 2



1 out of 2

+ 1 mark for vertical stretch
E9 [coordinate point labelled incorrectly (9, 4)]

Determine the equation of the radical function represented by the graph.



Solution

$y = 2\sqrt{x-1} - 2$

1 mark for vertical stretch
1 mark for horizontal translation
1 mark for vertical translation

3 marks

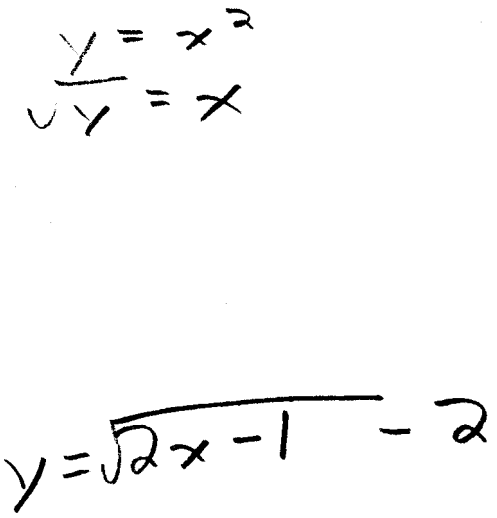
or

$y = \sqrt{4(x-1)} - 2$

1 mark for horizontal compression
1 mark for horizontal translation
1 mark for vertical translation

3 marks

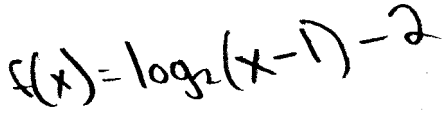
Exemplar 1



1 out of 3

+ 1 mark for vertical translation

Exemplar 2



1 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation
− 1 mark for concept error (logarithmic function instead of radical function)

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Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
14	D	P1
15	A	T2
16	D	R13
17	B	T4
18	B	P4
19	D	R12
20	B	R7
21	A	R4
22	C	R7, R8

Question 14 P1

There are 2 types of pencils, 3 colours of highlighters, and 5 styles of pens.

If you must select one of each to form a set, how many different sets of writing instruments are possible?

a) 10

b) 11

c) 25

d) 30

Question 15 T2

The point $P(\theta)$ lies on the unit circle. What are the coordinates of the point P if $\theta = 120^\circ$?

a) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Question 16 R13

Identify the function that has a domain of $\{x|x \geq 7\}$ and a range of $\{y|y \geq 0\}$.

a) $f(x) = \sqrt{x} + 7$

b) $f(x) = \sqrt{x} - 7$

c) $f(x) = \sqrt{x+7}$

d) $f(x) = \sqrt{x-7}$

Question 17 T4

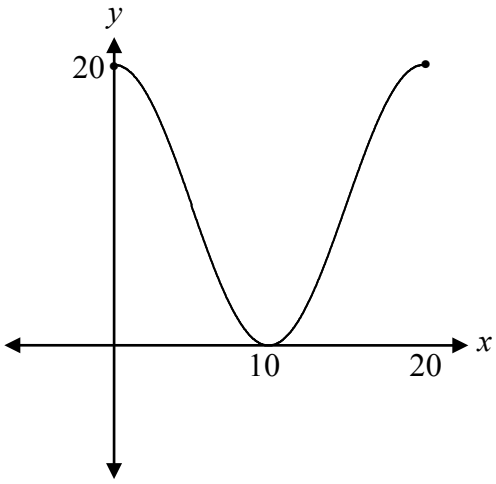
Using $y = -10\cos[B(x - C)] + D$, the value of C that corresponds to the following graph is:

a) 5

b) 10

c) 15

d) 20



Question 18 P4

Given the following row of Pascal's Triangle, identify the binomial expansion with these coefficients.

1 5 10 10 5 1

a) $(x+y)^4$

b) $(x+y)^5$

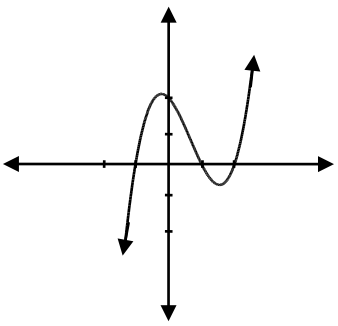
c) $(x+y)^6$

d) $(x+y)^7$

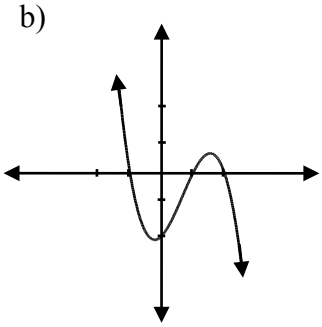
Question 19 R12

Identify the graph of the function $f(x) = -(x-2)(x-1)^2(x+1)$.

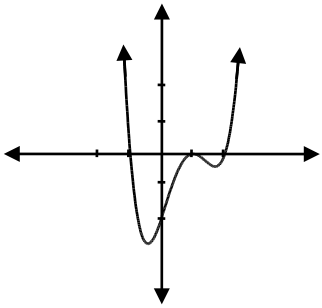
a)



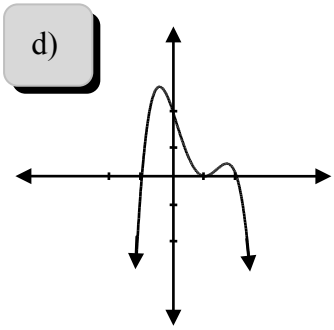
b)



c)



d)



Question 20 R7

Determine the value of $\log_9\left(\log_3 27\right)$.

a) $\frac{1}{3}$

b) $\frac{1}{2}$

c) 2

d) 3

Question 21 R4

If $(x,\ y)$ is a point on the graph of $y = f(x)$, identify the coordinates of this point on the graph of $g(x) = f(2x) + 5$.

a) $\left(\frac{x}{2},\ y+5\right)$

b) $(2x,\ y+5)$

c) $\left(\frac{x}{2},\ y-5\right)$

d) $\left(\frac{x}{2}-5,\ y\right)$

Question 22 R7, R8

Identify an equivalent expression for $1 + \log_2 5$

a) $\log_2 5$

b) $\log_2 7$

c) $\log_2 10$

d) $\log_2 11$

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Solve:

$$2\log_4 x - \log_4(x+3) = 1$$

Solution

$$2\log_4 x - \log_4(x+3) = 1$$

$$\log_4\left(\frac{x^2}{x+3}\right) = 1$$ 1 mark for power rule; 1 mark for quotient rule

$$4^1 = \frac{x^2}{x+3}$$ 1 mark for exponential form

$$4x + 12 = x^2$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$x = 6$ ~~$x = -2$~~ ½ mark for solving for x; ½ mark for rejecting extraneous root

4 marks

Exemplar 1

$$2\log_4\left(\frac{x}{x+3}\right)=1$$

$$\log_4\left(\frac{x}{x+3}\right)=\frac{1}{2}$$

$$\frac{x}{x+3}=4^{\frac{1}{2}}$$

$$\frac{x}{x+3}=2$$

$$x=2(x+3)$$

$$x=2x+6$$

$$x=-6$$

2½ out of 4

+ 1 mark for quotient rule
+ 1 mark for exponential form
+ ½ mark for solving for x

$8\log_4\left(\frac{x}{x+3}\right)^2 = 1$

$4^1 = \left(\frac{x}{x+3}\right)^2$

$4 = \frac{x^2}{(x^2+6x+9)}$

$4x^2 + 24x + 36 = x^2$

$3x^2 + 24x + 36$

$3(x^2 + 8x + 12)$

$+12$
$+8$
$6, 2$

$3(x+6)(x+2)$

$x = -6$ $\quad x = -2$

$\log_4(-6+3)$

$= \log_4(-3) = \text{und.}$

$\log_4(-2+3) = \log_4(1)$ ✓

$\log_4(-2)^2 = \log_4(4)$ ✓

2½ out of 4

+ 1 mark for quotient rule
+ 1 mark for exponential form
+ ½ mark for solving for *x*

E2 (changing an equation to an expression in lines 5 and 6)

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The following transformations are applied to $f(x)$, resulting in a new function, $g(x)$.

- reflection over the *y*-axis
- horizontal translation of 3 units to the right
- vertical translation of 4 units down

Write the equation of $g(x)$ in terms of $f(x)$.

Solution

$g(x) = f(-(x-3)) - 4$

1 mark for horizontal shift
1 mark for vertical shift
1 mark for reflection

or

3 marks

$g(x) = f(-x+3) - 4$

Exemplar 1

$g(x) = f(-x-3) - 4$

2 out of 3

+ 1 mark for vertical shift
+ 1 mark for reflection

Exemplar 2

$g(x) = (-x - 3) - 4$

1 out of 3

+ 1 mark for vertical shift
+ 1 mark for reflection
– 1 mark for concept error [$g(x)$ not shown as a transformation of $f(x)$]

Exemplar 3

$g(x) = f-(x-3) - 4$

3 out of 3

award full marks
E7 (notation error)

The height of a bicycle pedal as the bicycle is moving at a constant speed can be represented by the following function:

$$h(t) = 15\cos\frac{2\pi}{5}t + 30$$

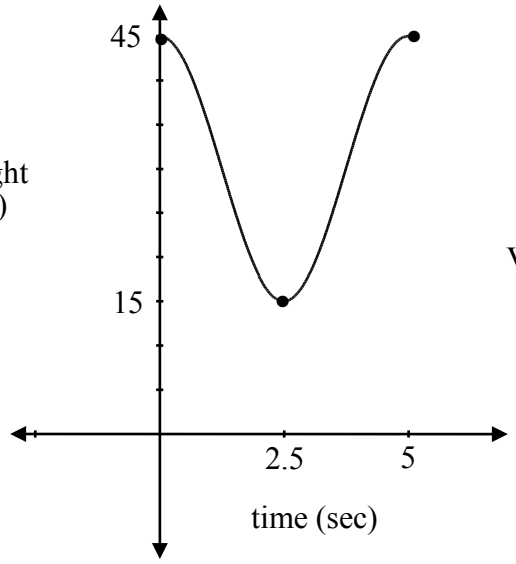
where h is the height of the pedal above the ground, in cm, and t is the time, in seconds.

a) Sketch a graph of at least one period of this function, where $t \geq 0$.

b) Determine the height of the bicycle pedal at 7.5 seconds.

Solution

a)



Amplitude = 15 — 1 mark for amplitude

$$\text{Period} = \frac{2\pi}{\frac{2\pi}{5}}$$ — 1 mark for period

$$= 5$$

Vertical Shift = 30 — 1 mark for vertical shift

3 marks

b) From the graph:

$h(t) = 15$ cm

1 mark

or

From the equation:

$$h(t) = 15\cos\frac{2\pi}{5}(7.5) + 30$$

$$= 15\cos 3\pi + 30$$

$$= 15(-1) + 30$$

$$= 15 \text{ cm}$$

Exemplar 1

a)

A = 15

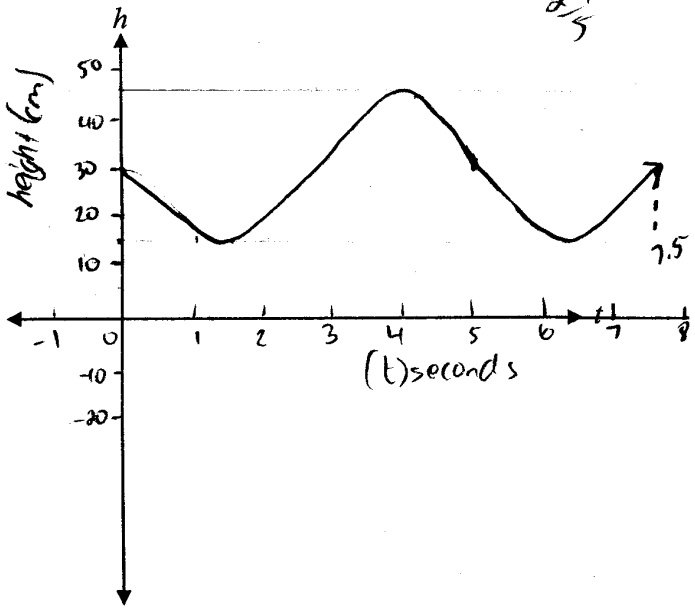
Midline = 30

$B = \frac{2\pi}{5}$

$\frac{2\pi}{\frac{2\pi}{5}} = P$ $2\pi \cdot \frac{5}{2\pi}$

$= \frac{10\pi}{2\pi}$

$P = 5$



2 out of 3

award full marks

– 1 mark for concept error (incorrect phase shift)

b)

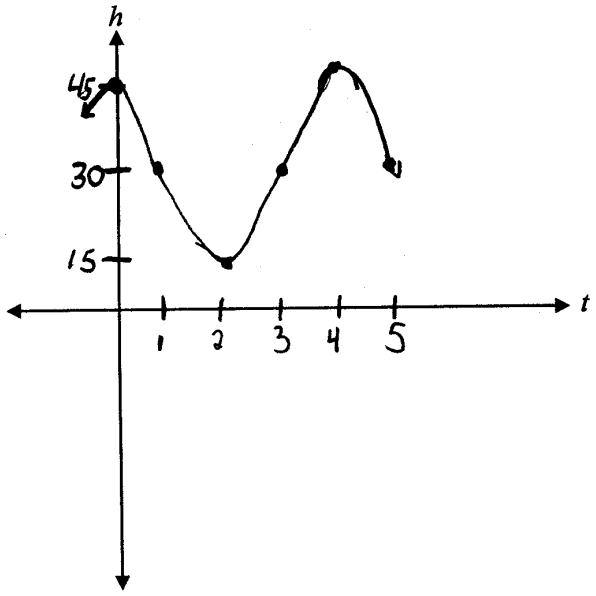
At 7.5 seconds the height is approximately 30cm

1 out of 1

work consistent with answer in a)

Exemplar 2

a)



2 out of 3

+ 1 mark for amplitude
+ 1 mark for vertical shit
E8 (answer included outside the given domain)

b)

$15\cos\left(\frac{2\pi}{5} * 7.5\right) + 30$

$15\cos\left(\frac{15\pi}{5}\right) + 30$

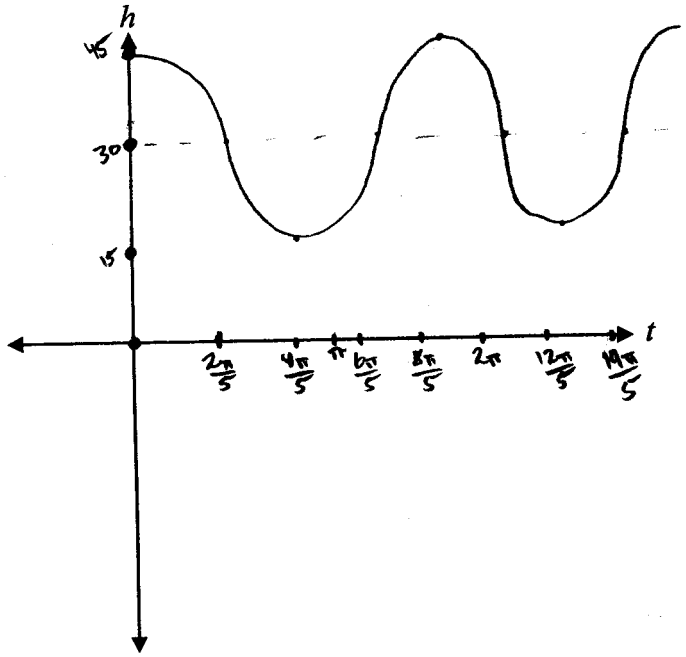
$15\cos(3\pi) + 30$

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 3

a)



2 out of 3

+ 1 mark for amplitude
+ 1 mark for vertical shift

b)

$$h(7.5) = 15\cos\frac{2\pi}{5}(7.5)+30$$

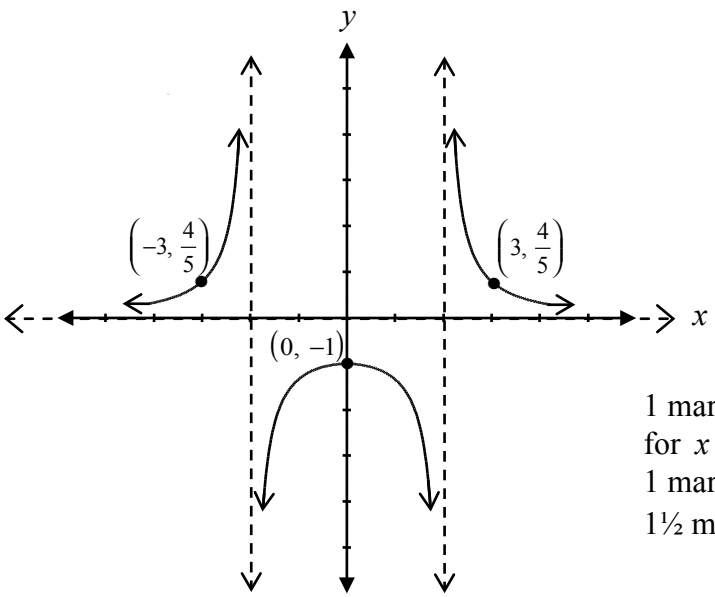
1 out of 1

award full marks
E1 (final answer not stated)

Sketch the graph of the function $f(x)$ and determine the y-intercept.

$$f(x) = \frac{4}{(x-2)(x+2)}$$

Solution



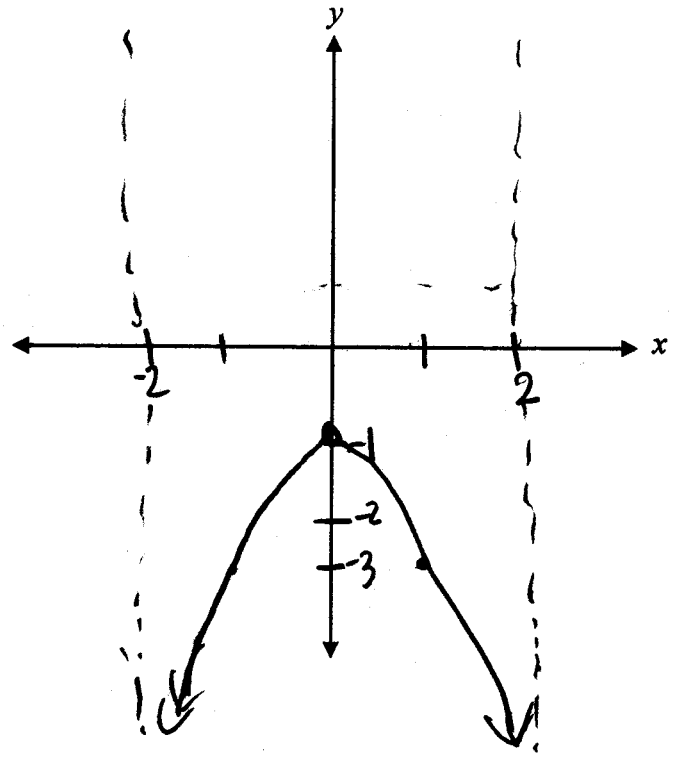
1 mark for vertical asymptotes (½ mark for $x = 2$, ½ mark for $x = -2$)
1 mark for horizontal asymptote at $y = 0$
1½ marks for shape (½ mark for shape in each section)

y-intercept: -1

½ mark for y-intercept

4 marks

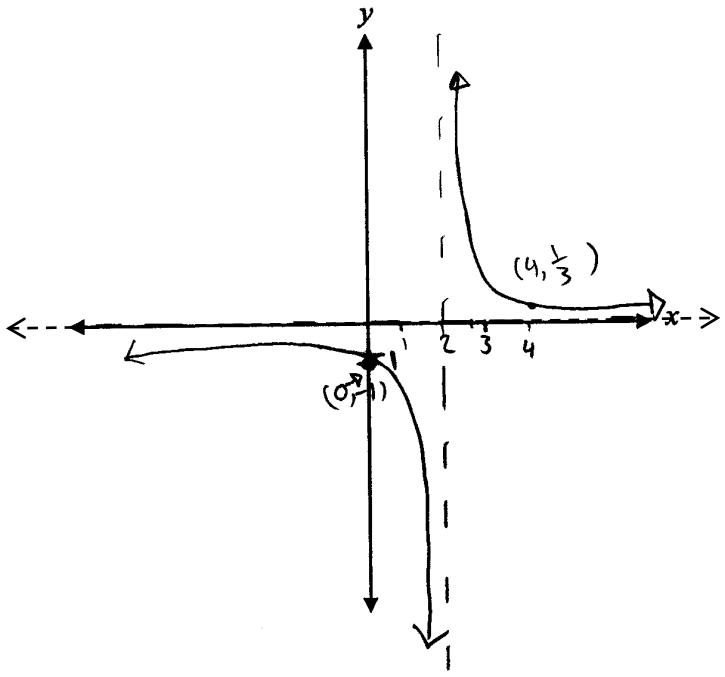
Exemplar 1



y-intercept: $x = -1$

1½ out of 4

+ 1 mark for vertical asymptotes
+ ½ mark for shape (between vertical asymptotes)
+ ½ mark for *y*-intercept
– ½ mark for arithmetic error (incorrect points on graph)



y-intercept: (0, -1)

yint

$y = \frac{4}{(0-2)(0+2)}$

$(-2)(+2)$

$y = \frac{4}{-4}$

$y = -1$

x	y
1	-4/3
4	1/3
5	

4/12

1/3

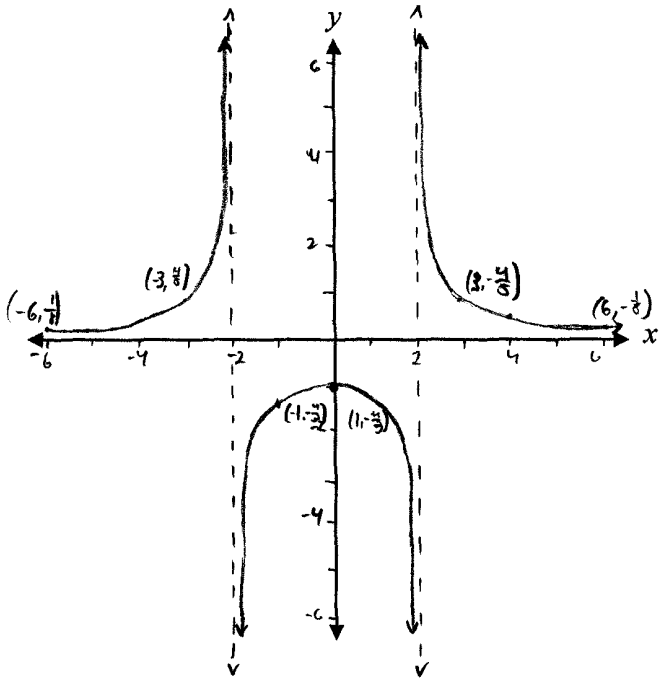
4/21

2½ out of 4

+ ½ mark for vertical asymptote at $x = 2$
+ 1 mark for horizontal asymptote at $y = 0$
+ ½ mark for shape to the right of asymptote at $x = 2$
+ ½ mark for y-intercept

Exemplar 3

$x \neq \pm 2$



x	y
-6	
-4	
-3	
-1	
0	-1
1	
3	
4	
6	

y-intercept: $y = -1$

3 out of 4

+ 1 mark for vertical asymptotes at $x = \pm 2$
+ 1½ marks for shape
+ ½ mark for y-intercept

E9 (incorrect or missing endpoints or arrowheads)

Kim solved the following logarithmic equation:

$$\log_2\left(-\frac{x}{3}\right) = \log_2(x-4)$$

$$-\frac{x}{3} = x - 4$$

$$-x = 3x - 12$$

$$-4x = -12$$

~~$x = 3$~~

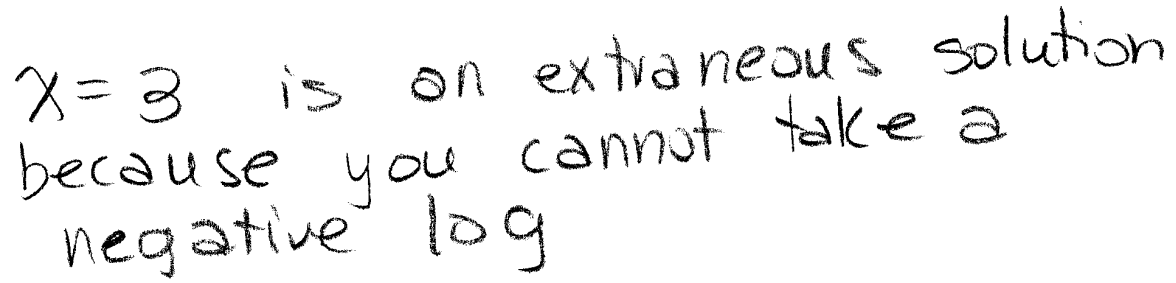
Explain why $x = 3$ is an extraneous solution.

Solution

$x = 3$ is an extraneous solution because the argument in a logarithmic equation cannot be negative.

1 mark

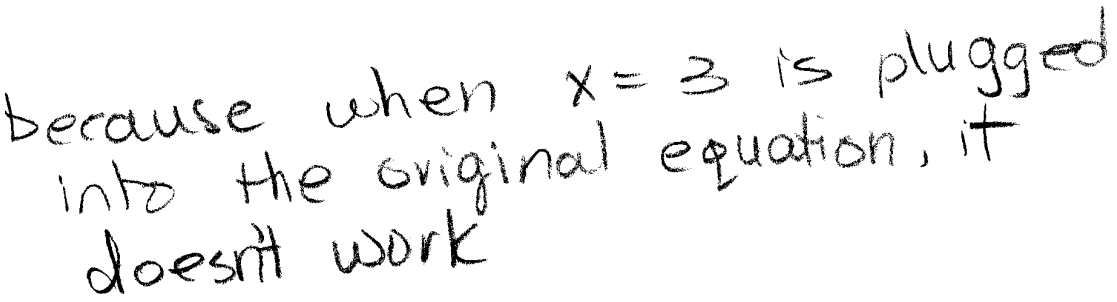
Exemplar 1



½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

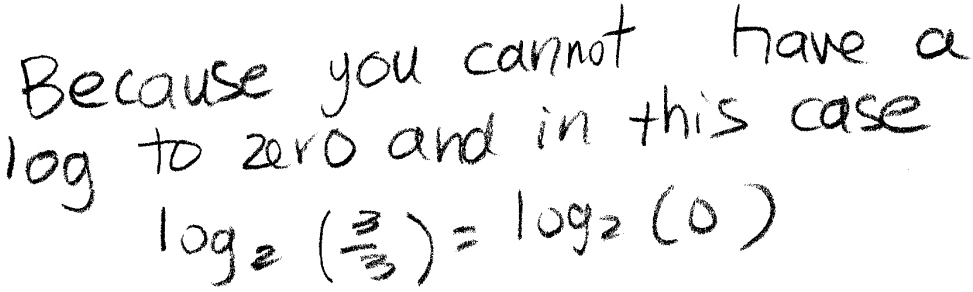
Exemplar 2



½ out of 1

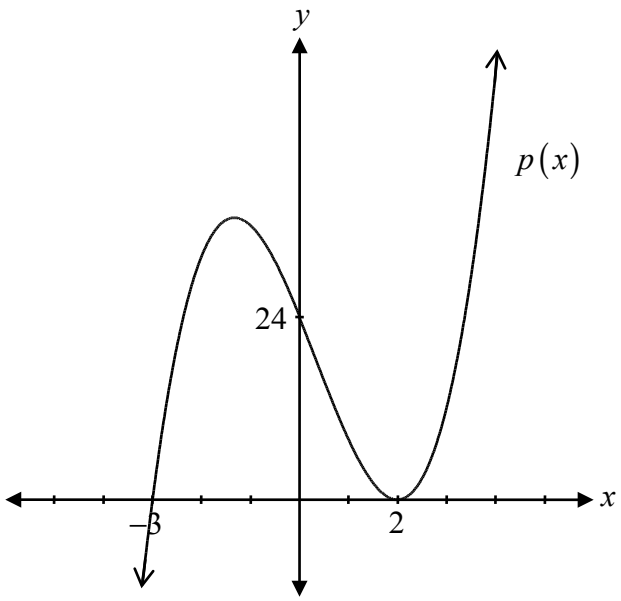
award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3



0 out of 1

Determine the equation of the polynomial function represented by the graph.



Solution

$p(x) = a(x-2)^2(x+3)$

$24 = a(0-2)^2(0+3)$

$24 = a(4)(3)$

$24 = 12a$

$a = 2$

$p(x) = 2(x-2)^2(x+3)$

1 mark for x-intercepts (½ mark for each)
1 mark for multiplicity at $x = 2$

1 mark for leading coefficient

3 marks

Exemplar 1

$p(x) = (x+3)(x-2)^2 + 24$

2 out of 3

+ 1 mark for *x*-intercepts
+ 1 mark for multiplicity

Exemplar 2

$y = (x+3)(x-2)$

$y = x^2 - 2x + 3x - 6$

$y = x^2 + x - 6$

1 out of 3

+ 1 mark for *x*-intercepts

Determine the coterminal angles with $\frac{2\pi}{3}$ over the interval $[-2\pi, 4\pi]$.

Solution

$-\frac{4\pi}{3}, \frac{8\pi}{3}$

1 mark (½ mark for each coterminal angle)

1 mark

Exemplar 1

$(-240^\circ, 120^\circ, 480^\circ)$

1 out of 1

award full marks
E5 (answer stated in degrees instead of radians)

Exemplar 2

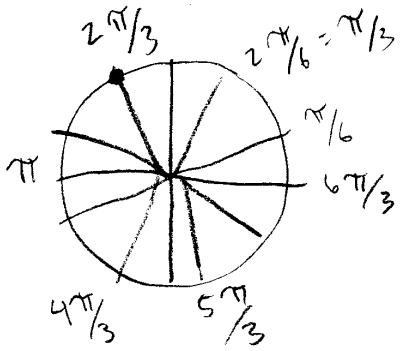
$\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$

$\frac{2\pi}{3} - 2\pi = \frac{-2\pi}{6}$

½ out of 1

award full marks
– ½ mark for arithmetic error

Exemplar 3



$\theta = \frac{8\pi}{3}, \frac{11\pi}{3}, -4\pi/3$

½ out of 1

award full marks
– ½ mark for procedural error (including one incorrect angle)

a) Solve the following equation:

$$0 = \sqrt{4x - 8} - 2$$

b) Explain how your answer in a) is related to the graph of $y = \sqrt{4x - 8} - 2$.

Solution

a) $4 = 4x - 8$

$12 = 4x$

$x = 3,\ x \geq 2$

1 mark

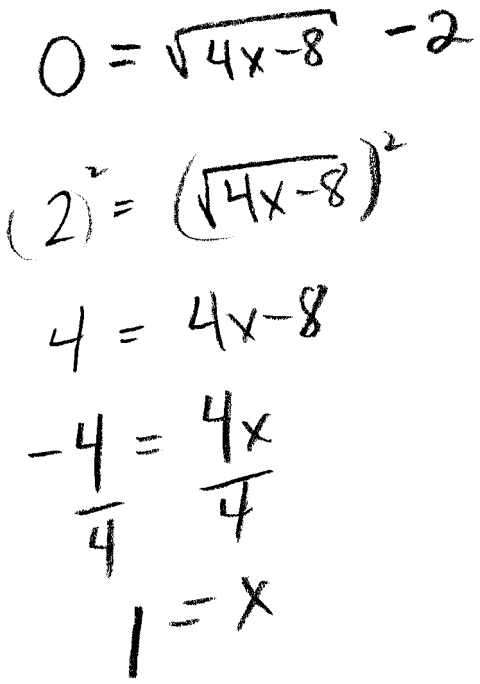
b) The answer in a) is the x-intercept of the graph. **1 mark**

Note(s):

- $x \geq 2$ does not need to be shown.

Exemplar 1

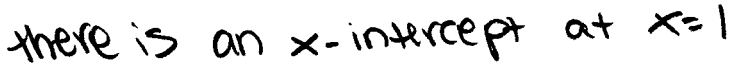
a)



½ out of 1

award full marks
– ½ mark for arithmetic errors in lines 4 and 5

b)



1 out of 1

work consistent with answer in a)

Exemplar 2

a)

$0 = \sqrt{4x-8} - 2$

$(2)^2 = \left(\sqrt{4x-8}\right)^2$

$4 = 4x - 8$

$\frac{12}{4} = \frac{4x}{4}$

$3 = x$

1 out of 1

b)

It's the same equation just rearranged to equal zero, when graphed this would shift the graph down since ∅ is it's new base line

0 out of 1

Exemplar 3

a)

$0 = \sqrt{4x-8} - 2$

$4x - 8 \geq 0$

$x \geq 2$

$2^2 = 4x - 8$

$4 = 4x - 8$

$12 = 4x$

$x = 3$ ✓

1 out of 1

b)

my answer is the solution of the graph $y = \sqrt{4x-8} - 2$

0 out of 1

Determine the exact value of $\sin\frac{13\pi}{12}$.

Solution

$$\sin\frac{13\pi}{12} = \sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$$

1 mark for combination

$$= \left(\sin\frac{3\pi}{4}\right)\left(\cos\frac{\pi}{3}\right)+\left(\cos\frac{3\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

2 marks for exact values (½ mark for each)

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

3 marks

Note(s):

- Other combinations are possible.

Exemplar 1

$\sin \frac{13\pi}{12}$

$\sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right)$

$\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$\sin\frac{3\pi}{4} \cdot \cos\frac{\pi}{3} + \cos\frac{3\pi}{4} \cdot \sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

1½ out of 3

+ 1 mark for combination
+ ½ mark for an exact value

Exemplar 2

$\frac{9\pi}{12} + \frac{4\pi}{12}$

$\left(\underset{A}{\frac{3\pi}{4}}\right) + \left(\underset{B}{\frac{\pi}{3}}\right)$

$\sin(a+b) = \sin\left(\frac{\sqrt{2}}{2}\right)\cos\left(\frac{1}{2}\right) + \cos\left(-\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{-\sqrt{6}}{4}$

$\sin\left(\frac{13\pi}{12}\right) = \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

3 out of 3

award full marks
E7 (notation error in line 3)

Given the functions $f(x) = x + 2$ and $g(x) = \dfrac{1}{x-5}$:

a) Determine the equation of the composite function $f(g(x))$ and its domain.

b) Determine the x-intercept and y-intercept of $f(g(x))$.

Solution

a) $f(g(x)) = \dfrac{1}{x-5} + 2$

or

$f(g(x)) = \dfrac{2x-9}{x-5}$

domain: $\{x \in \mathbb{R},\ x \neq 5\}$

1 mark for composition

1 mark for domain

2 marks

b) x-intercept: $\dfrac{9}{2}$

y-intercept: $\dfrac{9}{5}$

½ mark for x-intercept

½ mark for y-intercept

1 mark

Exemplar 1

a)

$$y = (x+2)\left(\frac{1}{x-5}\right)$$

$$= \frac{(x+2)}{(x-5)}$$

$f(g(x)) =$ $\frac{(x+2)}{(x-5)}$

domain: $(-\infty, 5) \sim (5, \infty)$

1 out of 2

+ 1 mark for domain
E7 (notation error in domain)

b)

x-intercept: $x = -2$

y-intercept: $y = -\frac{2}{5}$

$\frac{2}{-5}$

1 out of 1

work consistent with answer in a)

Exemplar 2

a)

$\frac{1}{x-5} + 2$

$\frac{1}{x-5} + \frac{2x-10}{x-5}$

$\frac{2x-9}{x-5}$

$\frac{x-9}{x-5}$

$f(g(x)) = \frac{x-9}{x-5}$

domain: $(-\infty, \infty)$

1 out of 2

+ 1 mark for composition
E7 (transcription error in line 4)

b)

x-intercept: 9, 5

y-intercept: $9/5 = 1\,4/5$

½ out of 1

+ ½ mark for y-intercept

Exemplar 3

a)

$f\left(\frac{1}{x-5}\right)$

$\frac{1}{x+2-5}$

$\frac{1}{x-3}$

$f(g(x)) = \frac{1}{x-3}$

domain: $(x \neq 3, x \in R)$

1 out of 2

+ 1 mark for domain consistent with $f(g(x))$

b)

$0 = \frac{1}{x-3}$

x-intercept: nothing

y-intercept: $-\frac{1}{3}$

$y = \frac{1}{0-3}$

1 out of 1

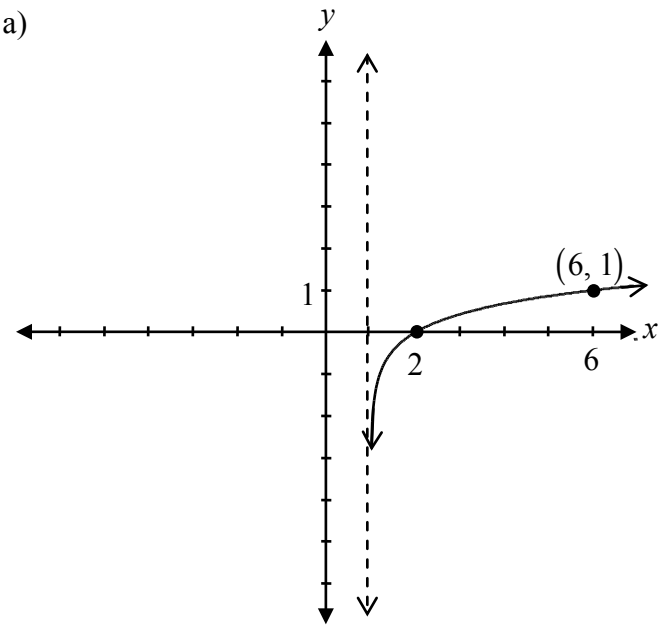
work consistent with answer in a)

a) Sketch the graph of $f(x) = \log_5(x-1)$.

b) Sketch the graph of $f^{-1}(x)$.

Solution

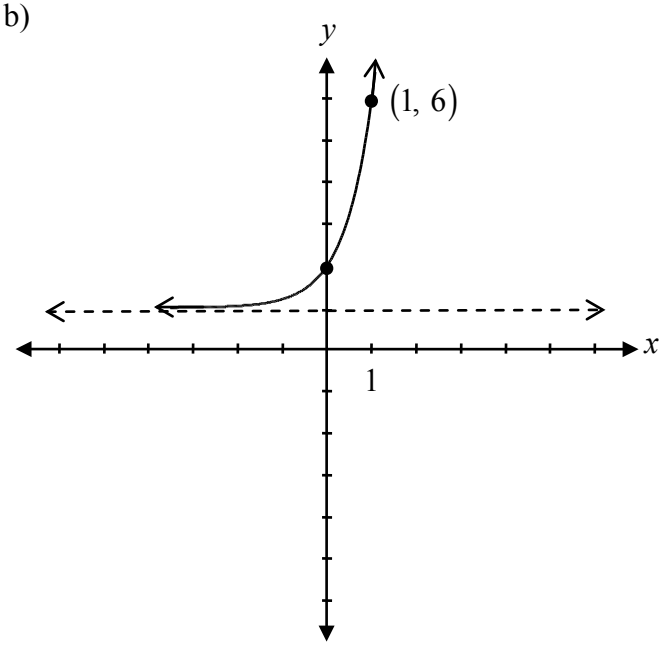
a)



½ mark for vertical asymptote at $x = 1$
½ mark for x-intercept at $x = 2$
½ mark for increasing logarithmic function
½ mark for consistent point on the logarithmic graph

2 marks

b)

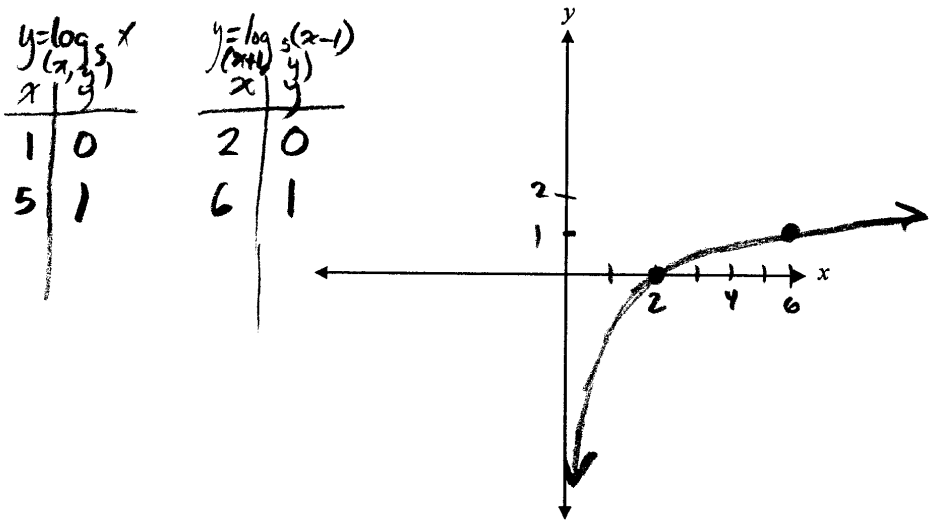


1 mark for the graph of the inverse function consistent with a)

1 mark

Exemplar 1

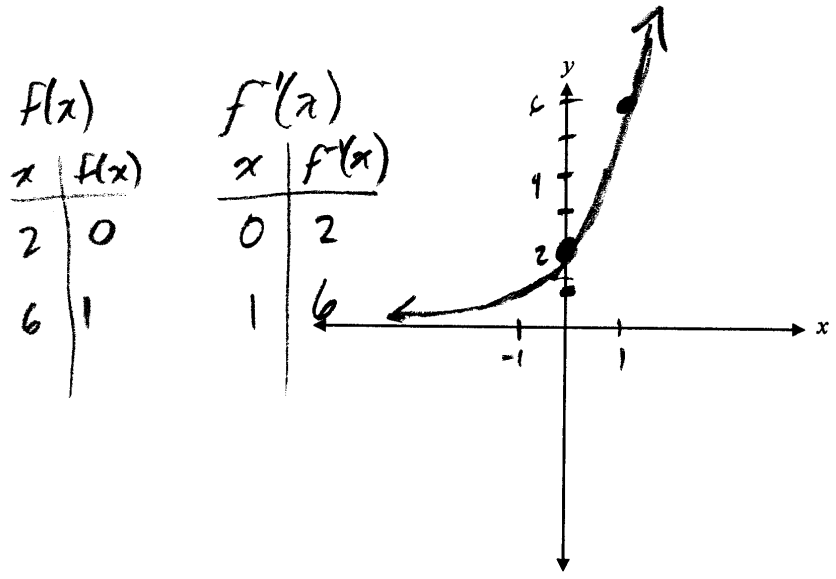
a)



1½ out of 2

+ ½ mark for x-intercept at $x = 2$
+ ½ mark for increasing logarithmic function
+ ½ mark for consistent point on the graph

b)

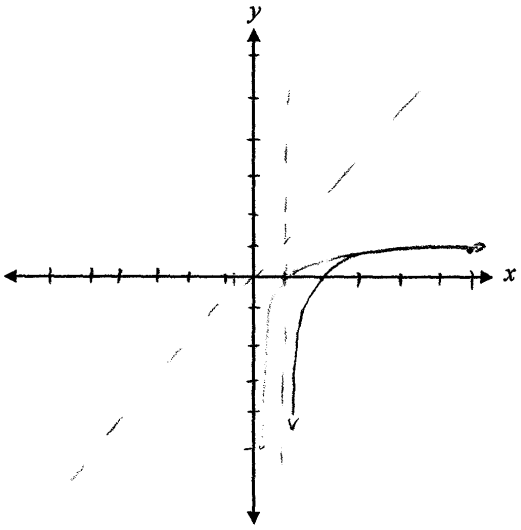


1 out of 1

work consistent with answer in a)

Exemplar 2

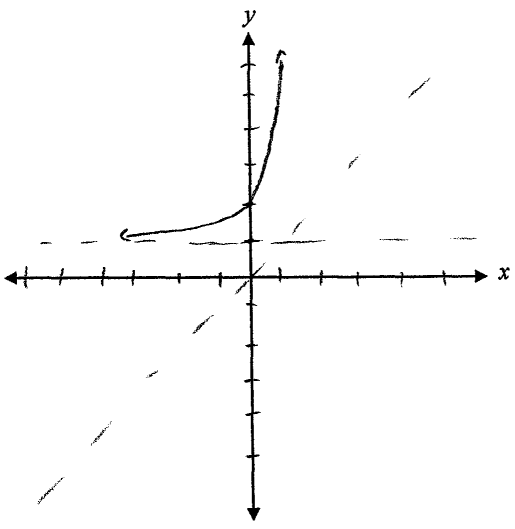
a)



2 out of 2

award full marks
E9 (scale values on axes not indicated)

b)



$y = \log_5 (x-1)$

$5^y = (x-1)$

$5^x = y-1$

1 out of 1

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Question 34 R14

Explain what the graph of a rational function looks like near a vertical asymptote.

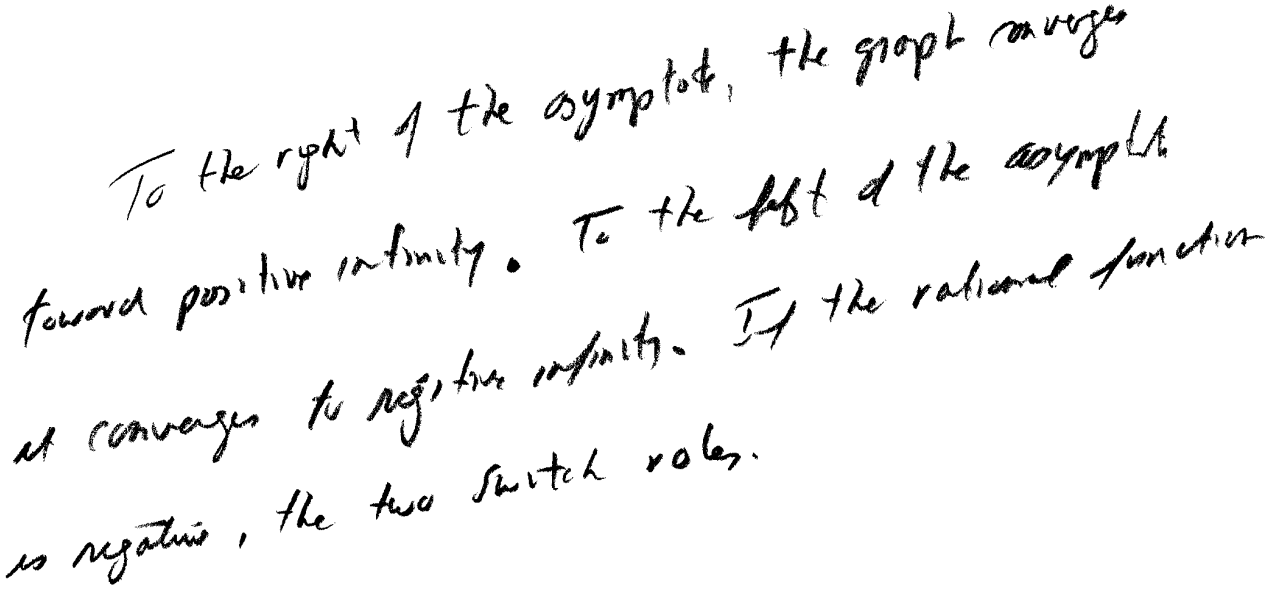
Solution

The graph approaches infinity (positive or negative) as it approaches the asymptote. **1 mark**

or

The graph approaches the asymptote, but does not touch it.

Exemplar 1



½ out of 1

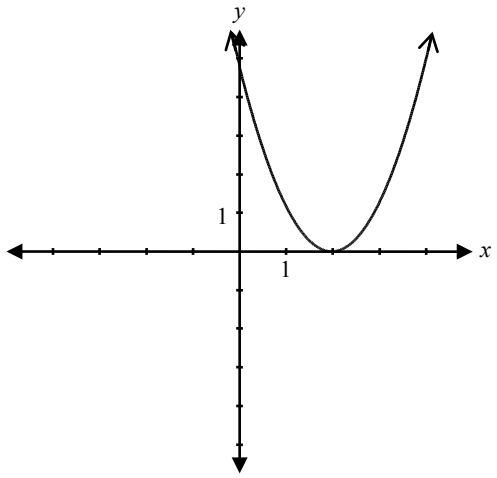
award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

½ out of 1

award full marks
– ½ mark for terminology error in explanation

Given the graph of $f(x) = (x-2)^2$,



determine one possible restriction for the domain of $f(x)$ so that its inverse is a function.

Solution

Domain: $[2, \infty)$

1 mark

or

Domain: $(-\infty, 2)$

Note(s):

- Other solutions are possible.

Exemplar 1

Domain: $\{x \in R \mid x \leq 0\}$

1 out of 1

Exemplar 2

Domain: $x \neq +2$

0 out of 1

Over the interval $[0,\ 2\pi]$, determine the non-permissible values of θ in the expression $\csc\theta(\cos\theta+1)$.

Solution

$\csc\theta = \dfrac{1}{\sin\theta}$	1 mark for correct substitution of an appropriate identity
$\sin\theta \neq 0$	½ mark for $\sin\theta \neq 0$
$\theta = 0,\ \pi,\ 2\pi$	½ mark for consistent non-permissible values

2 marks

Exemplar 1

$\cos\theta + 1 \neq 0$

$\cos\theta \neq -1$

$\theta = \pi$

½ out of 2

+ ½ mark for consistent value of θ

Exemplar 2

$\csc\theta = \frac{1}{\cos\theta}$

$\cos\theta \neq 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

1 out of 2

+ ½ mark for $\cos\theta \neq 0$ consistent with line 1
+ ½ mark for consistent non-permissible values

Exemplar 3

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta \neq 0$

$\theta = 0, \pi$

1½ out of 2

+ 1 mark for correct substitution of an appropriate identity
+ ½ mark for $\sin\theta \neq 0$

Explain why ${}_3C_8$ is undefined.

Solution

In the formula ${}_nC_r$, the number of objects, *n*, must be larger than or equal to the number of objects selected, *r*.

1 mark

or

You can't select 8 objects from a total of 3.

Exemplar 1

3 cannot make 8 groups

½ out of 1

award full marks
– ½ mark for terminology error in explanation

Exemplar 2

because the bigger number is suppose to go first. ($\therefore {}_8C_3$.)

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

because you can't do the factorial of a negative number

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Solve:

$${}_nP_3 = 48(n-1)$$

Solution

$$\frac{n!}{(n-3)!} = 48(n-1)$$

½ mark for substitution in correct formula

$$\frac{(n)(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 48(n-1)$$

1 mark for expansion of factorial
½ mark for simplification of factorials

$$n(n-2) = 48$$

$$n^2 - 2n - 48 = 0$$

$$(n-8)(n+6) = 0$$

$$n = 8 \quad \cancel{n = -6}$$

½ mark for solving for both values of n
½ mark for rejecting extraneous solution

3 marks

Exemplar 1

$$\frac{n!}{(n-3)!} = 48n-48$$

$$n\cdot(n-1)(n-2) = 48n-48$$

$$n(n^2-3n+2) = 48n-48$$

$$n^3-3n^2+2n-48n+48=0$$

$$n^3-3n^2-46n+48=0$$

2 out of 3

+ ½ mark for substitution
+ 1 mark for expansion
+ ½ mark for simplification of factorials

Exemplar 2

$$\frac{n!}{n.(n-3!)} = 48(n-1)$$

$$\frac{(n)(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

$$\frac{(n)(n-1)(n-2)}{(n-1)} = \frac{48(n-1)}{(n-1)}$$

$$(n)(n-2) = 48$$

$$n = 8$$

2 out of 3

+ ½ mark for substitution
+ 1 mark for expansion
+ ½ mark for simplification of factorials

E7 (transcription error in line 1)
E2 (changing an equation to an expression in line 2)

Christine dives off a diving board.

Her dive is modelled by the function $h(t) = t^3 - 3t^2 - t + 3$, where h is her height in metres, relative to the water surface and t is the time in seconds after diving off the diving board.

a) Given that $(t+1)$ is a factor for the function $h(t)$, determine the other factors.

b) Sketch the graph of the function $h(t)$ for the time interval $t = 0$ to $t = 3$.

c) Determine how many seconds Christine is underwater.

Solution

a)
$$\begin{array}{r|cccc} -1 & 1 & -3 & -1 & 3 \\ & \downarrow & -1 & 4 & -3 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$h(t) = (t+1)\left(t^2 - 4t + 3\right)$

$h(t) = (t+1)(t-1)(t-3)$

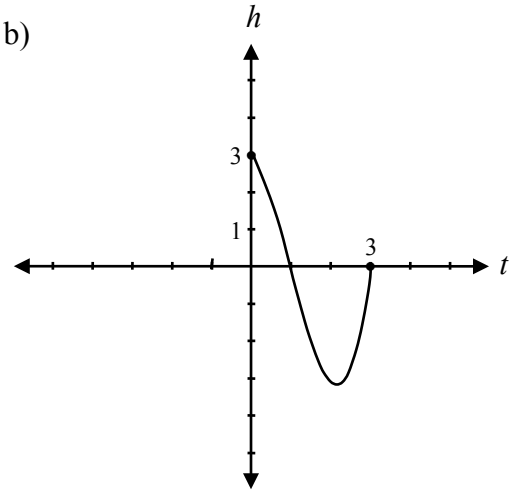
½ mark for $t = -1$

1 mark for synthetic division (or equivalent strategy)

½ mark for determining other factors

2 marks

b)



½ mark for t-intercepts
½ mark for h-intercept

1 mark

c) Christine is underwater for 2 seconds.

1 mark

Exemplar 1

a)

$$\begin{array}{r|cccc} -1 & 1 & -3 & -1 & +3 \\ & \downarrow & -1 & 4 & -3 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$(t+1)$

$x^2 - 4x + 3$

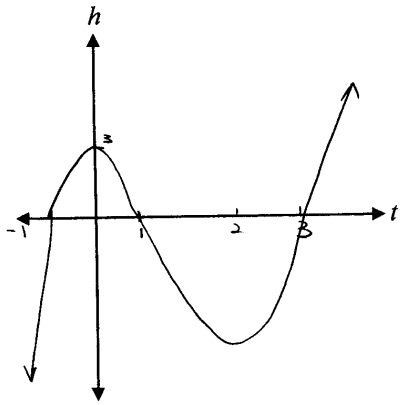
$= (x-3)(x-1)$

All factors are $(t+1)(t-3)(t-1)$

2 out of 2

award full marks
E3 (variable introduced without being defined)

b)



1 out of 1

award full marks
E8 (answer included outside the given domain)

c)

She was under water from
1 second to 3 seconds

1 out of 1

award full marks
E1 (final answer not stated)

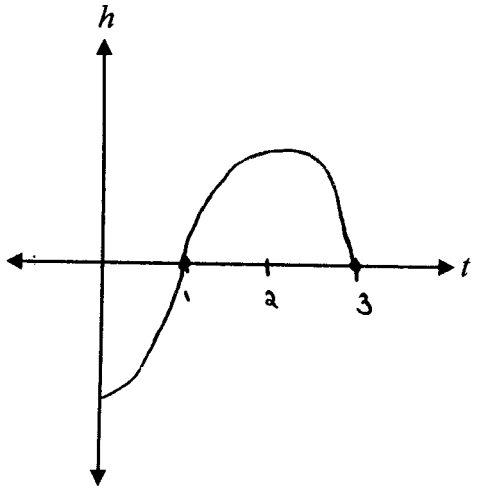
Exemplar 2

a)

$h(t) = (t+1)(t-1)(t-3)$

2 out of 2

b)



½ out of 1

+ ½ mark for *t*-intercepts

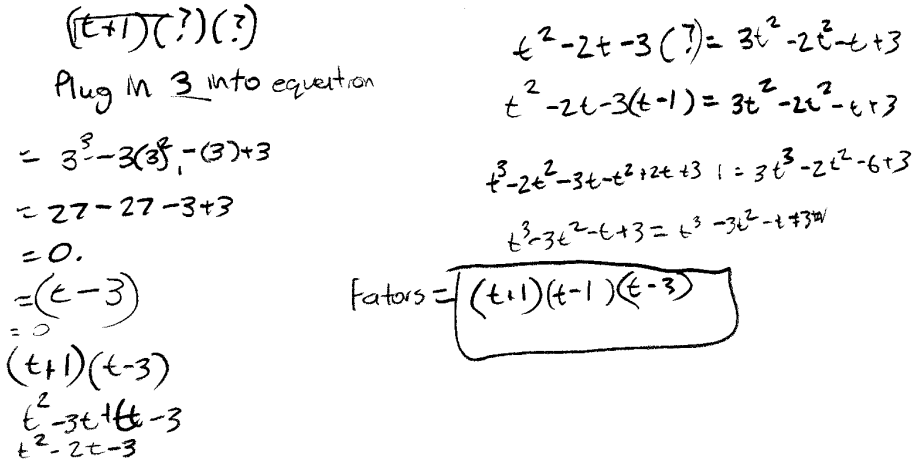
c)

She was under water for 1 second.

1 out of 1

Exemplar 3

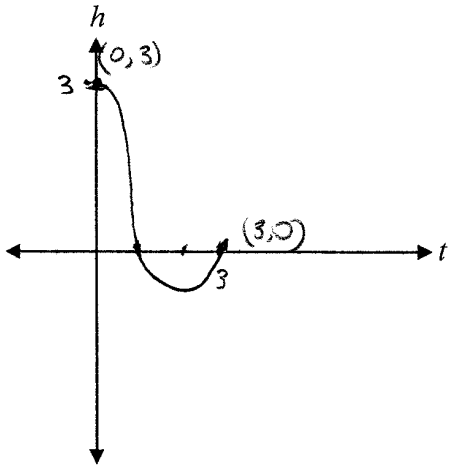
a)



2 out of 2

award full marks
E4 (missing brackets but still implied in line 2)
E7 (notation error in line 7)

b)



1 out of 1

c)

Christine was under water for 2 seconds

1 out of 1

Question 40 T6

Prove the identity for all permissible values of x.

$$\sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\sec x + \tan x$	
$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$	1 mark for correct substitution of appropriate identities
$\frac{(1+\sin x)(1-\sin x)}{\cos x(1-\sin x)}$	1 mark for logical process to prove the identity
$\frac{1-\sin^2 x}{(1-\sin x)\cos x}$	
$\frac{\cos^2 x}{(1-\sin x)\cos x}$	1 mark for algebraic strategies
$\frac{\cos x}{1-\sin x}$	**3 marks**

Method 2

$$\sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

Left-Hand Side	Right-Hand Side	
	$\frac{\cos x}{1 - \sin x}$	
	$\frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$	1 mark for logical process to prove the identity
	$\frac{\cos x(1 + \sin x)}{1 - \sin^2 x}$	
	$\frac{\cos x(1 + \sin x)}{\cos^2 x}$	1 mark for correct substitution of appropriate identities
	$\frac{1 + \sin x}{\cos x}$	
	$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$	1 mark for algebraic strategies
	$\sec x + \tan x$	**3 marks**

Exemplar 1

Left-Hand Side	Right-Hand Side
$\sec x + \tan x$ $\frac{1}{\cos x} + \frac{\sin x}{\cos x}$ $\frac{1+\sin x}{\cos x}$	$\frac{\cos x}{1-\sin x}$

1 out of 3

+ 1 mark for correct substitution of appropriate identities

Exemplar 2

Left-Hand Side	Right-Hand Side
$\frac{1}{\cos x} + \frac{\cos x}{\sin x}$ $\frac{1}{\cos x}\left(\frac{\sin x}{\sin x}\right) + \frac{\cos x(\cos x)}{\sin x(\cos x)}$ $\frac{\sin x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}$ $\frac{\sin x + \cos^2 x}{\cos x \sin x}$ $\frac{\sin x + 1 - \sin^2 x}{\cos x \sin x}$ $\frac{-\sin^2 x + \sin x + 1}{\cos x \sin x}$ $\frac{-1(\sin^2 x - \sin x - 1)}{\cos x \sin x}$	$\frac{\cos x}{1-\sin x}$

1 out of 3

+ 1 mark for algebraic strategies

Exemplar 3

Left-Hand Side	Right-Hand Side
$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$	$\frac{\cos x}{1-\sin x}$
$= \frac{1+\sin x}{\cos x}\left(\frac{\cos x}{\cos x}\right)$	
$= \frac{(1+\sin x)(\cos x)}{\cos^2 x}$	
$= \frac{\cos x + \sin x \cos x}{1-2\sin^2 x}$	
$= \frac{1\cos x}{1-2\sin x}$	
$= \frac{\cos x}{1-\sin x}$	

1 out of 3

+ 1 mark for logical process to prove the identity

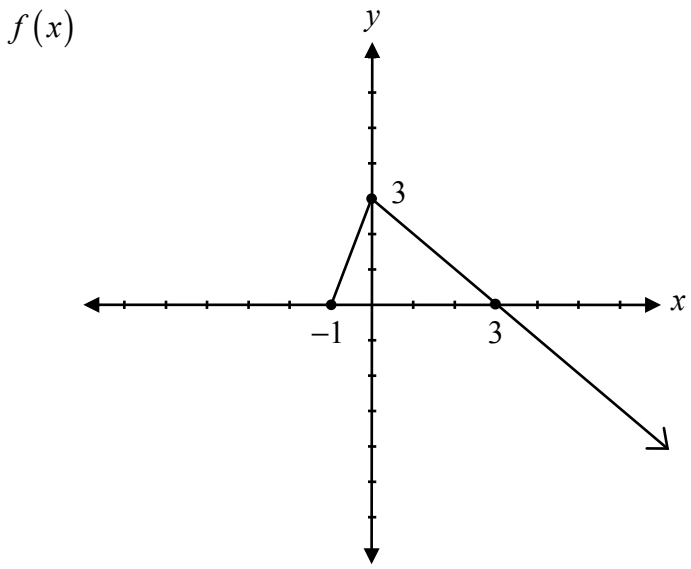
Exemplar 4

Left-Hand Side	Right-Hand Side
$\sec x + \tan x$	$\frac{\cos x}{1-\sin x}\frac{(1+\sin x)}{(1+\sin x)}$
$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$	$= \frac{\cos x + \sin x}{1-\sin^2 x}$
$= \frac{\sin x + 1}{\cos x}\left(\frac{\cos x}{\cos x}\right)$	$= \frac{\cos x + \sin x}{\cos^2 x}$
$= \frac{\sin x + \cos x}{\cos^2 x}$	

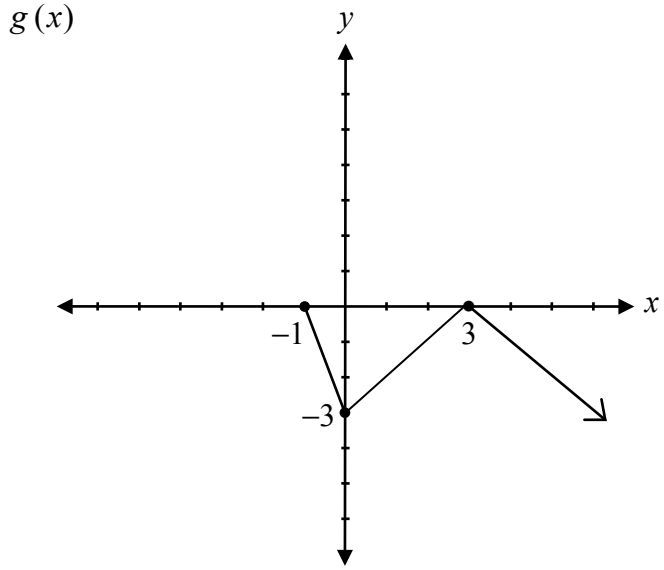
2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for logical process to prove the identity

Given the graph of $f(x)$, sketch the graph of the function $g(x) = -|f(x)|$.



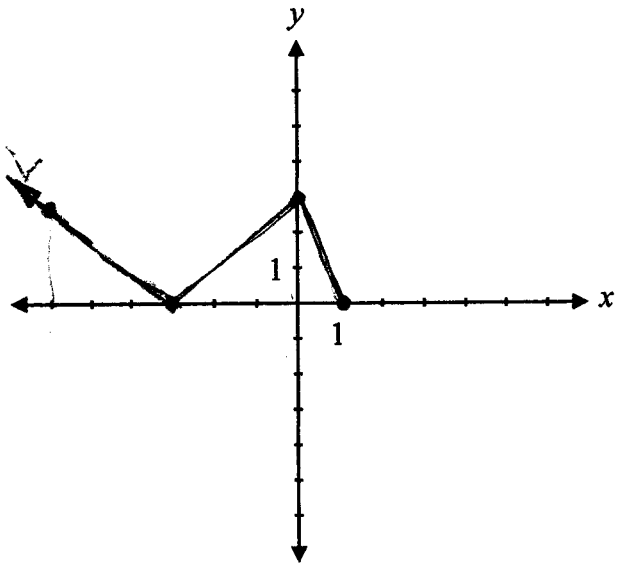
Solution



1 mark for absolute value
1 mark for vertical reflection

2 marks

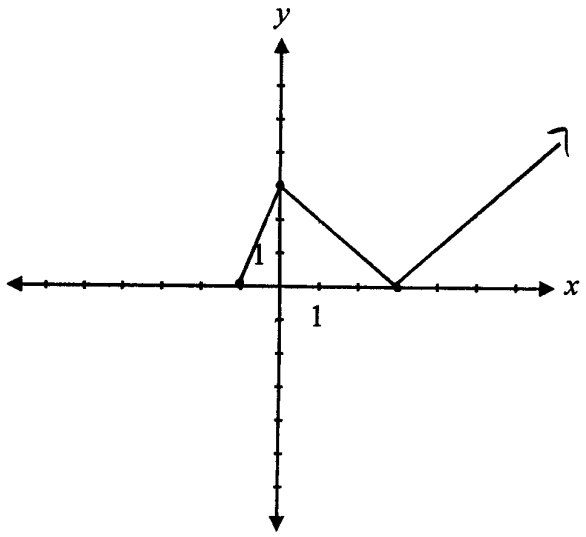
Exemplar 1



1 out of 2

+ 1 mark for absolute value

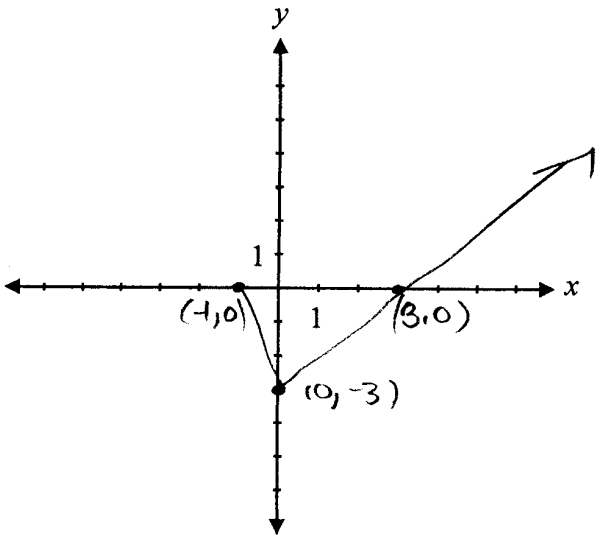
Exemplar 2



1 out of 2

+ 1 mark for absolute value

Exemplar 3



1 out of 2

+ 1 mark for vertical reflection

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Given that $\cot\theta = -\frac{2}{5}$, where θ is in Quadrant IV, determine the exact value of $\sin\theta$.

Solution

$P(2, -5)$

$r^2 = (2)^2 + (-5)^2$ ½ mark for substitution of $x = 2,\ y = -5$

$r^2 = 4 + 25$ ½ mark for solving for r

$r = \sqrt{29}$

$\sin\theta = \frac{-5}{\sqrt{29}}$ 1 mark for $\sin\theta$ (½ mark for quadrant, ½ mark for value)

2 marks

Exemplar 1

$$\cot\theta = -\frac{2}{5} = \frac{x}{y}$$

$$x^2 + y^2 = r^2 \Rightarrow 2^2 + 5^2 = r^2$$

$$\Rightarrow 4 + 20 = r^2$$

$$\Rightarrow \sqrt{24} = r \Rightarrow r = 2\sqrt{6}$$

S	A
T	C

$$\sin\theta = \frac{-5}{2\sqrt{6}}$$

1½ out of 2

+ ½ mark for substitution of $x = 2,\ y = 5$
+ 1 mark for consistent value of $\sin\theta$

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated
E2 equation/expression	▪ changing an equation to an expression ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer included outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

Problem(s) noted: _____

_____

_____

_____

_____

_____

Question(s) affected: _____

_____

_____

Action taken or rationale for assigning marks: _____

_____

_____

_____

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_____

Follow-up: _____

_____

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_____

_____

_____

Decision: _____

_____

_____

_____

_____

_____

_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
10	R5	1
12	R3	2
21	R4	1
24	R2, R5	3
32 a)	R1	2
32 b)	R1	1
33 b)	R6	1
35	R6	1
41	R1, R5	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
9	T1	1
15	T2	1
17	T4	1
25 a)	T4	3
25 b)	T4	1
29	T1	1
31	T3	2
36	T3	1
42	T2	2
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
4	P3	3
5	P4	3
14	P1	1
18	P4	1
37	P3	1
38	P2	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
8	R12	1
19	R12	1
28	R12	3
39 a)	R11	2
39 b)	R12	1
39 c)	R12	1

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
2	T5	4
7	T5	4
31	T6	1
36	T6	1
40	T6	3
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R8, R10	4
6	R10	2
20	R7	1
22	R7, R8	1
23	R10	4
27	R10	1
33 a)	R9	2
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
11	R14	1
13	R13	3
16	R13	1
26	R14	4
30 a)	R13	1
30 b)	R13	1
34	R14	1