Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2016

Manitoba

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Booklet 1. June 2016

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre (formerly the Manitoba Text Book Bureau). Order online at www.mtbb.mb.ca.

This resource will also be available on the Manitoba Education and Advanced Learning website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	–	14	34
Booklet 2	8	20	56
Total	8	34	**90**

* The first 5 questions in *Booklet 1* require a calculator. 
You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The perforated pages can be removed from the test booklet. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

Instructions

- There are 14 questions worth a total of 34 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Formula Sheet

$s = \theta r$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$

$\log_a(M^n) = n\log_a M$

$P(n,r)$ or ${}_nP_r = \dfrac{n!}{(n-r)!}$

$C(n,r)$ or ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

$t_{k+1} = {}_nC_k a^{n-k}b^k$

For $ax^2 + bx + c = 0$,

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*.
These words are defined below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Solve: Give a solution for a problem or determine the value(s) of a variable.

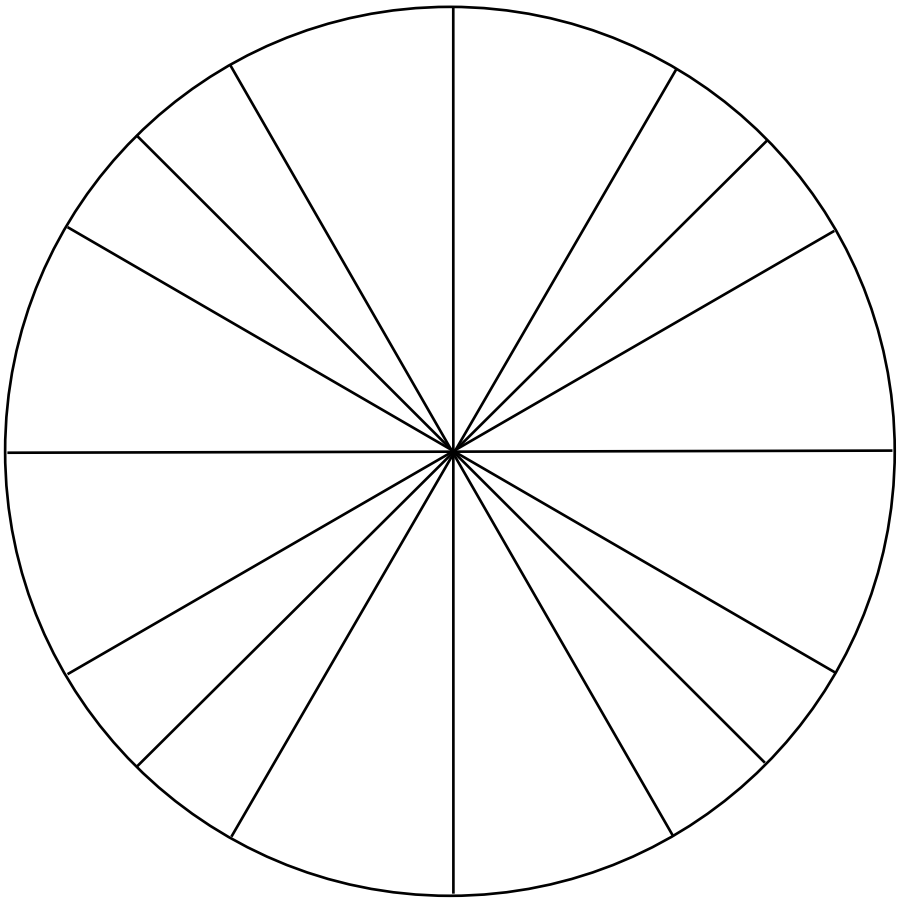
Verify: Establish the truth of a statement by substitution or comparison.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

State: Give an answer without an explanation or justification.

Describe: Use words to provide the process or to report details of the response.

No marks will be awarded for work done on this page.



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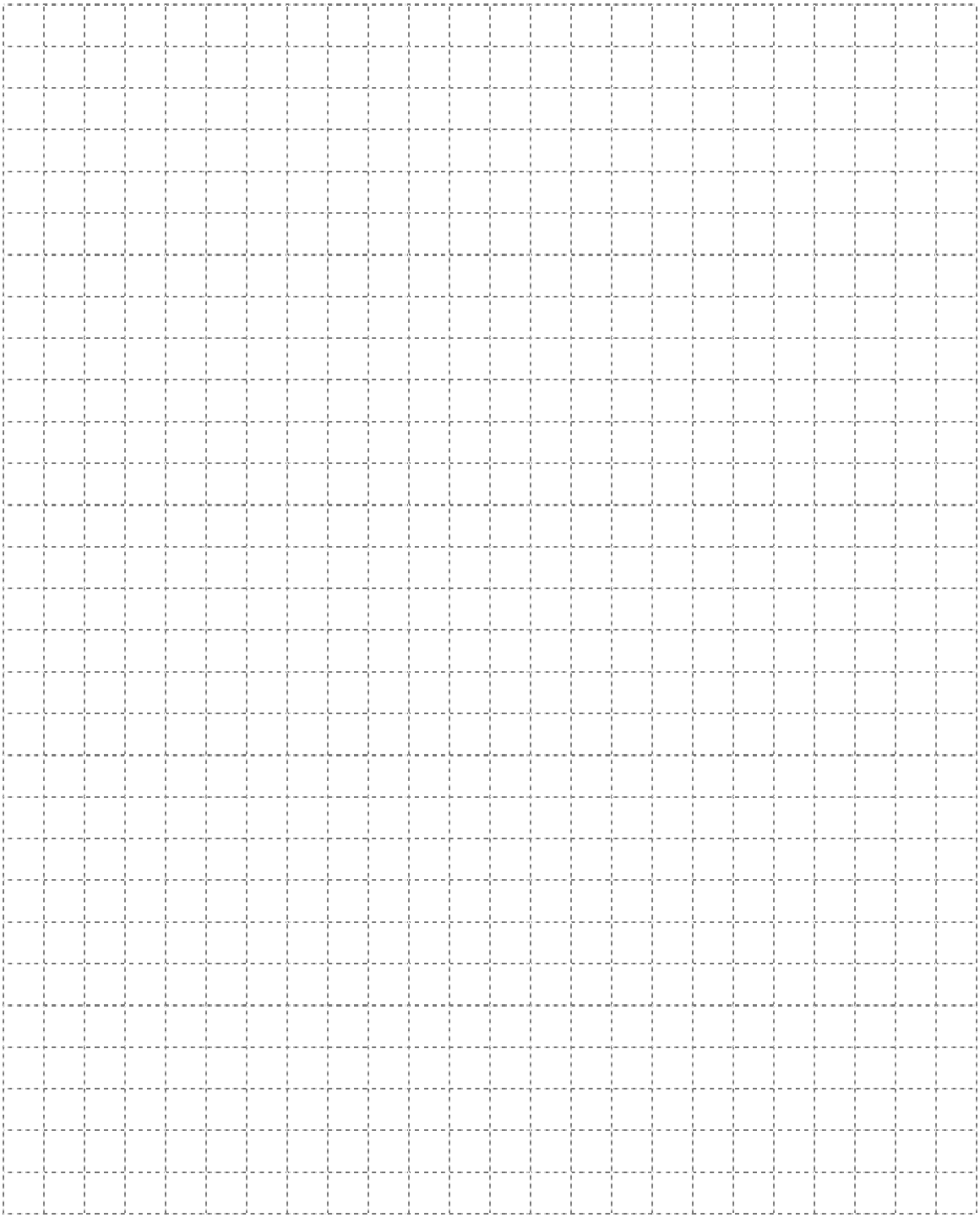


Question 1  2 marks 101

A wheel has a diameter of 20 cm and rotates through a central angle of 252°.

Determine how far the wheel rolled.

Question 2  3 marks 102

Solve the following equation over the interval $[0,\ 2\pi]$:

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

Question 3  3 marks 103

Determine and simplify the fourth term in the expansion of $\left(2x^4 - 3y\right)^8$.

Question 4  3 marks 104

Sheeva's bank is lending her $50 000 at an annual interest rate of 6%, compounded monthly, to purchase a car.

Given that the last payment will be a partial payment, determine how many full monthly payments of $800 Sheeva will have to make.

The formula below may be used.

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

where $PV =$ the present value of the amount borrowed

$R =$ the amount of each periodic payment

$$i = \frac{\text{annual interest rate (as a decimal)}}{\text{the number of compounding periods per year}}$$

$n =$ the number of equal periodic payments

Express your answer as a whole number.

Question 5  2 marks 105

An employee asked 10 people in an ice cream shop to wait in line.

Determine the number of different arrangements possible if two of the people, Jamie and John, refused to stand next to each other in the line.

Note: A calculator is not required for the remaining test questions.

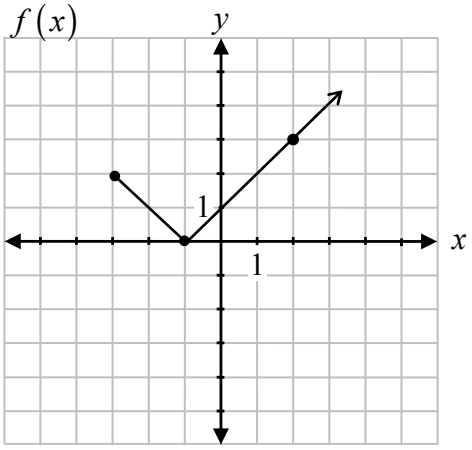
Question 6 — 1 mark — 106

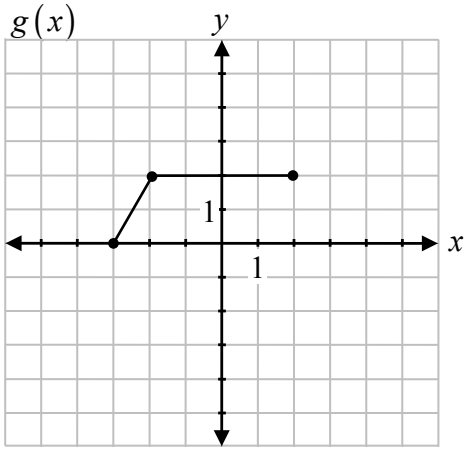
The point $(-2, 4)$ is on the graph of $f(x)$.

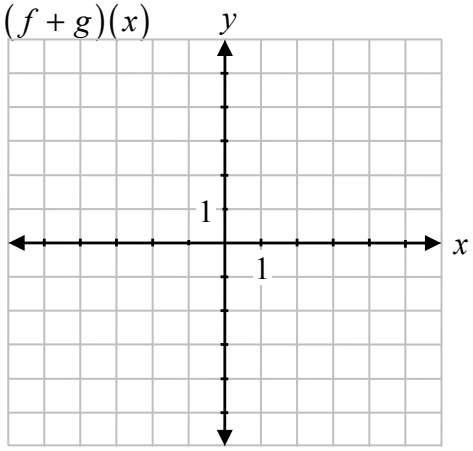
State the coordinates of the corresponding point when $f(x)$ is reflected over the y-axis.

Question 7 — 2 marks — 107

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f+g)(x)$.







Question 8 — 3 marks — 108

Using the laws of logarithms, fully expand the expression:

$$\log_2\left(\frac{w^3 x}{y-1}\right)$$

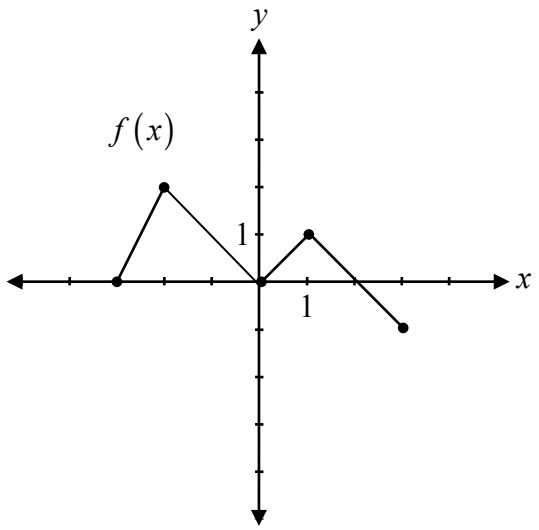
Question 9 — 4 marks — 109

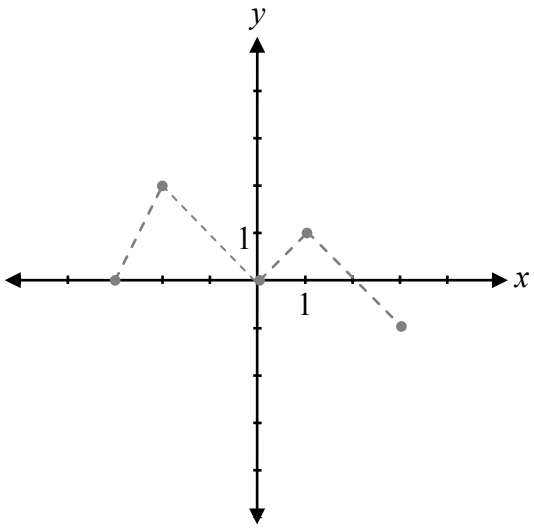
Solve the following equation algebraically for θ, where $0 \leq \theta \leq 2\pi$:

$$2\cos 2\theta = 1$$

Question 10 — 3 marks — 110

Given the graph of $y = f(x)$, sketch the graph of $y = 2\left|f(x-1)\right|$.





The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 11 — 3 marks — 111

Prove the identity for all permissible values of θ:

$$\cos\theta + \tan\theta\sin\theta = \frac{\tan\theta\sin\theta}{1-\cos^2\theta}$$

Left-Hand Side	Right-Hand Side

Question 12 — 1 mark — 112

Raoul has 8 shirts, 5 pairs of pants, and 3 hats. He adds the options together and determines that he has 16 different outfits to wear.

Raoul made an error in calculating the number of different outfits. Describe how to determine the correct number of outfits.

Question 13

a) 1 mark b) 1 mark

113
114

Given $f(x) = 2x - 1$ and $g(x) = x^2 + 1$:

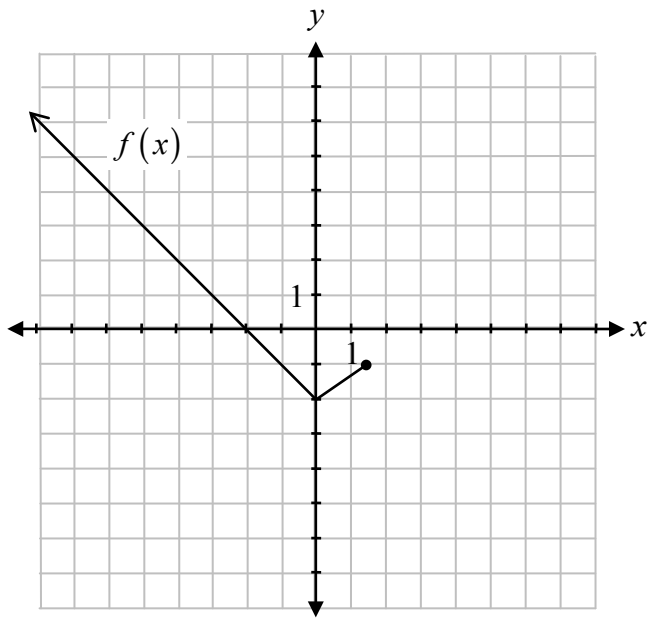
a) Determine $f(x) \bullet g(x)$.

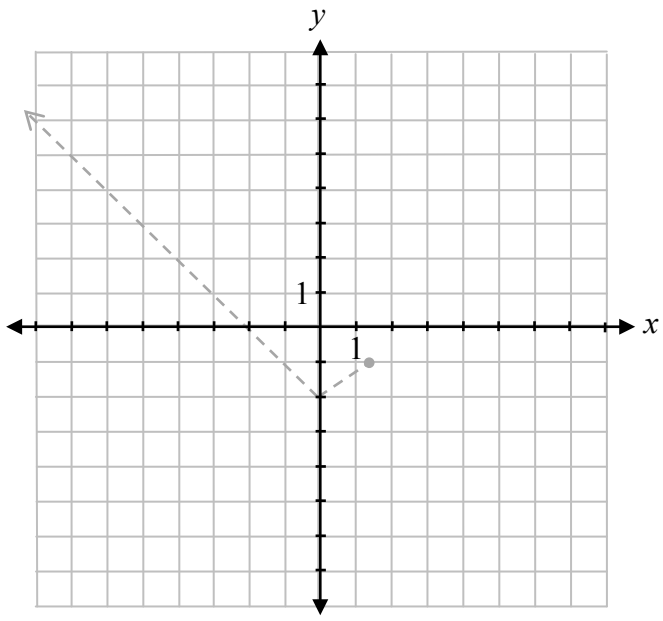
b) Determine $g(g(x))$.

Question 14 — 2 marks

115

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.





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