Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2019

Manitoba

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, ext. 7590
Email: youyi.sun@gov.mb.ca

Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error). **Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur)**				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

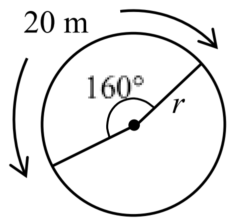
Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 T1

Determine the length of the radius, r, given an arc length of 20 metres and a central angle of 160°.



Solution

$$\theta = (160)\left(\frac{\pi}{180}\right)$$
$$= \frac{8\pi}{9}$$

1 mark for conversion

$$r = \frac{s}{\theta}$$
$$r = \frac{20}{\left(\frac{8\pi}{9}\right)}$$

1 mark for substitution

$$r = \frac{180}{8\pi}\ \text{m}$$

2 marks

or

$$r = 7.162\,\text{m}$$

Exemplar 1

$$160 \times \frac{\pi}{180}$$

$$= \frac{8\pi}{9}$$

$$\frac{20}{\frac{8\pi}{9}} = \frac{\frac{8\pi}{9} r}{\frac{8\pi}{9}}$$

$$20 \times \frac{9}{8\pi} = r$$

$$\boxed{70.685 = r}$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 5
E5 (units of measure omitted in final answer)

Exemplar 2

$$S = \theta r$$

$$20m = 160 r$$

$$\frac{20m}{160} = 0.125 m$$

1 out of 2

+ 1 mark for substitution
E1 (final answer not stated)

Exemplar 3

$S = \theta r$

$20 = 160r$

$20 = \frac{2\pi}{3} r$

$\boxed{r = 9.5493 \text{ m}}$

1 out of 2

+ 1 mark for substitution

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Question 2 P1

There are eight cars parked in a row. Determine the number of possible arrangements of these eight cars if Mrs. Jones must always park in the third spot and Mr. Rodriguez must always park in the last spot.

Solution

$\underline{6} \bullet \underline{5} \bullet \underline{1} \bullet \underline{4} \bullet \underline{3} \bullet \underline{2} \bullet \underline{1} \bullet \underline{1}$

720 **1 mark**

Exemplar 1

$\underline{6}\ \underline{5}\ \underline{1}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1}$

${}_6P_6 + {}_1P_1 + {}_1P_1$

$= 720 + 1 + 1$

$= 722$

0 out of 1

Exemplar 2

$\underline{8}\ \underline{7}\ \underline{1}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{1}$

$= 20160$ ways

0 out of 1

Exemplar 3

$6!$

1 out of 1

award full marks
E1 (final answer not stated)

Bill wins \$1 300 000 in a lottery and invests the entire amount at an annual interest rate of 2.5% compounded quarterly. He will withdraw \$10 000 at the end of every three months.

Determine, algebraically, the total number of withdrawals, including the partial amount, that Bill can make until there is no money left. Express your answer as a whole number.

Use the formula:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

where PV = the present value deposited

R = the amount of each withdrawal

n = the number of equal withdrawals

$i = \dfrac{\text{the annual interest rate (in decimal form)}}{\text{the number of compounding periods}}$

Solution

$$1300\,000 = \frac{10\,000\left[1-\left(1+\frac{0.025}{4}\right)^{-n}\right]}{\frac{0.025}{4}}$$

½ mark for substitution

$$8125 = 10\,000\left[1-(1.00625)^{-n}\right]$$

$$0.8125 = 1-(1.00625)^{-n}$$

$$-0.1875 = -(1.00625)^{-n}$$

½ mark for simplification

$$\log(0.1875) = -n\log 1.00625$$

½ mark for applying logarithms
1 mark for power law

$$-\frac{\log(0.1875)}{\log(1.00625)} = n$$

$$268.672\,348 = n$$

½ mark for solving for n

Bill can make 269 withdrawals.

3 marks

Exemplar 1

$$\$1300\,000 = \frac{\$10000[1-(1+0.00625)^{-n}]}{0.00625} \qquad i = 0.025/4 = 0.00625$$

$$\$8125 = \$10000[1-(1.00625)^{-n}]$$

$$0.8125 = 1-(1.00625)^{-n}$$

$$-0.1875 = -(1.00625)^{-n}$$

$$-0.1875 = -(-n\log 1.00625)$$

$$-0.1875 = n(-\log 1.00625)$$

$$\frac{-0.1875}{-\log 1.00625} = n$$

$$69.293 = n$$

Bill needs 69 equal withdrawals and then a partial withdrawal until there is no more money remaining.

2 out of 3

+ ½ mark for substitution
+ ½ mark for simplification
+ 1 mark for power law
+ ½ mark for solving for *n*
– ½ mark for arithmetic error in line 6

Exemplar 2

$$1300\,000 = \frac{10\,000\left(1-\left(1+\frac{0.025}{4}\right)^{-n}\right)}{\frac{0.025}{4}}$$

$$\frac{1300\,000}{0.00625} = \left(\frac{10\,000\left(1-(1.00625)^{-n}\right)}{0.00625}\right) 0.00625$$

$$\frac{8125}{10\,000} = \frac{10\,000\left(1-(1.00625)^{-n}\right)}{10\,000}$$

$$-1 \quad 0.8125 = \left(1-(1.00625)^{-n}\right. \quad -1$$

$$-1\cdot(-1.8125) = \left(-(1.00625)^{-n}\right)\cdot -1$$

$$1.8125 = (1.00625)^{-n}$$

$$\log_{1.00625}(1.8125) = -n$$

$$95.45018201 = -n$$

$$-95.450 = n$$

2½ out of 3

+ ½ mark for substitution
+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for solving for *n*

E1 (impossible solution not rejected in final answer)

Exemplar 3

$$(0.0083)\ 1300000 = \frac{10000\left[1-(1+0.0083)^{-n}\right]}{\cancel{0.0083}\ \cancel{(0.0083)}}$$

$$\frac{10790}{10000} = \frac{\cancel{10000}\left[1-(1.0083)^{-n}\right]}{\cancel{10000}}$$

$$\frac{1.079}{1} = \frac{\cancel{1}-(1.0083)^{-n}}{\cancel{1}}$$

$$1.079 = 1.0083^{-n}$$

$$\frac{\log 1.079}{\log 1.0083} = \frac{-n \log \cancel{1.0083}}{\log \cancel{1.0083}}$$

$$\frac{-n}{-1} = \frac{9.199}{-1}$$

$$n = -9.199$$

$\therefore$ He can make 10 withdrawals until there is no money remaining

---

2 out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for solving for *n*
E1 (impossible solution not rejected in final answer)

Determine and simplify the 12th term in the binomial expansion of $\left(x^3 - \frac{1}{2x^2}\right)^{12}$.

Solution

$$t_{12} = {}_{12}C_{11}\left(x^3\right)^1\left(-\frac{1}{2x^2}\right)^{11}$$

2 marks (1 mark for ${}_{12}C_{11}$; ½ mark for each consistent factor)

$$t_{12} = (12)\left(x^3\right)\left(-\frac{1}{2048x^{22}}\right)$$

$$t_{12} = -\frac{12}{2048x^{19}}$$

1 mark for simplification (½ mark for coefficient; ½ mark for exponent)

or

$$t_{12} = -\frac{3}{512x^{19}}$$

or

$$t_{12} = -0.006x^{-19}$$

3 marks

Exemplar 1

$t_{k+1} = {}_nC_k(a)^{n-k}(b)^k$ k is one less than solving for

$\therefore \quad k = 11$

$t_{12} = {}_{12}C_{11}(x^3)^1\left(-\frac{1}{2x^2}\right)^{11}$

$= (12)(x^3)\left(-\frac{1}{2048x^{22}}\right)$

$= \frac{12x^3}{2048x^{22}}$

$= \frac{3x^3}{512x^{22}}$

2 out of 3

+ 1 mark for ${}_{12}C_{11}$

+ 1 mark for consistent factors

Exemplar 2

$({}_{12}C_{12})(x^3)^0\left(\frac{-1}{2x^2}\right)^{12} = (1)(1)\left(\frac{1}{4096x^{24}}\right)$

$= \boxed{\frac{1}{4096x^{24}}}$

2 out of 3

+ 1 mark for consistent factors

+ 1 mark for simplification

Exemplar 3

$$t_{12} = {}_{13}C_{12}(x^3)^{13-12}\left(-\frac{1}{2x^2}\right)^{12}$$

$$t_{12} = {}_{13}C_{12}(x^3)\left(\frac{1}{4096x^{24}}\right)$$

$$t_{12} = 12\,(x^3)\left(\frac{1}{4096x^{24}}\right)$$

$$t_{12} = 12\left(\frac{1}{4096x^{27}}\right)$$

$$t_{12} = \frac{1}{49152x^{27}}$$

1 out of 3

+ 1 mark for consistent factors

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Question 5 R10

Solve, algebraically.

$$e^{2x-3} = 7^{x+1}$$

Solution

$\ln e^{2x-3} = \ln 7^{x+1}$ ½ mark for applying logarithms

$(2x-3)\ln e = (x+1)\ln 7$ 1 mark for power law

$2x - 3 = x\ln 7 + \ln 7$

$2x - x\ln 7 = \ln 7 + 3$ ½ mark for collecting terms with x

$x(2 - \ln 7) = \ln 7 + 3$

$x = \dfrac{\ln 7 + 3}{2 - \ln 7}$ ½ mark for isolating x

$x = 91.438\ 783$ ½ mark for evaluating quotient of logarithms

$x = 91.439$

3 marks

Exemplar 1

$$(2x-3)\log e = (x+1)\log 7$$
$$2x\log e - 3\log e = x\log 7 + \log 7$$

1½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law

Exemplar 2

$$\ln e^{2x-3} = \ln 7^{x+1}$$
$$2x-3\ln e = x+1\ln 7$$
$$2x-3 = x\ln 7+\ln 7$$

1½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
E4 (missing brackets but still implied)

Exemplar 3

$$2x-3\ln e = x+1\ln 7$$
$$2x-x = \ln 7+3$$
$$x = \ln 7+3$$
$$x = 4.946$$

2½ out of 3

award full marks
– ½ mark for procedural error in line 1

Exemplar 4

$$2x-3\ln e = x+1\ln 7$$

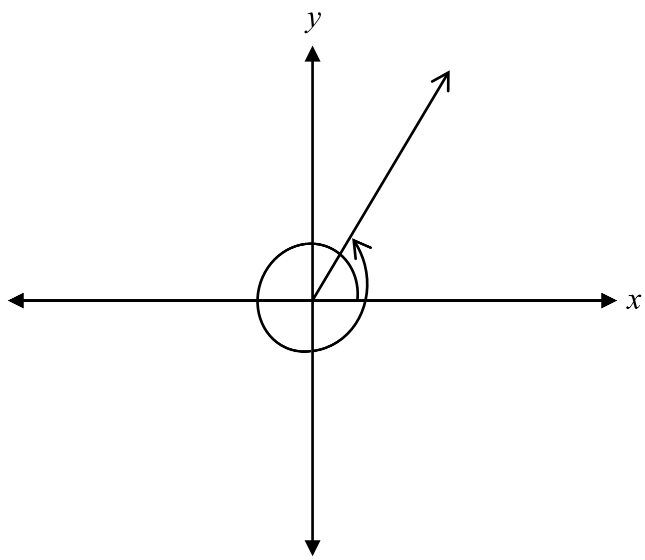
1 out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
– ½ mark for procedural error

Question 6 T1

Sketch the angle $\frac{7\pi}{3}$ in standard position.

Solution



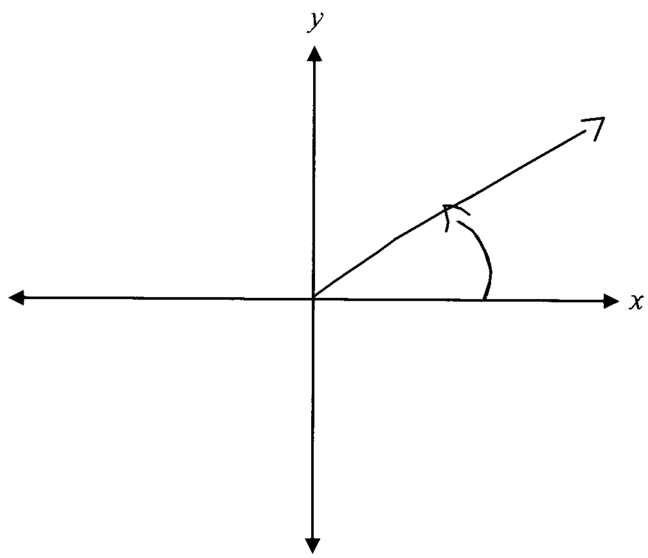
½ mark for an appropriate angle in quadrant I
½ mark for correct number of revolutions

1 mark

Note:

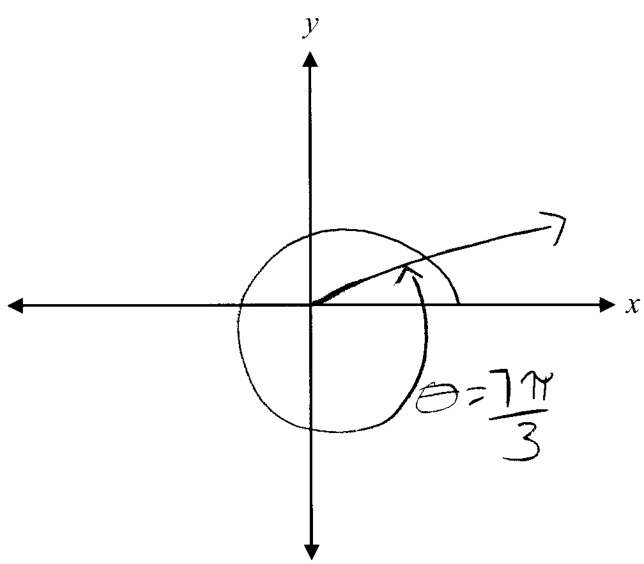
- If the directional arrow is not indicated, deduct an E1 error (final answer not stated).

Exemplar 1



0 out of 1

Exemplar 2

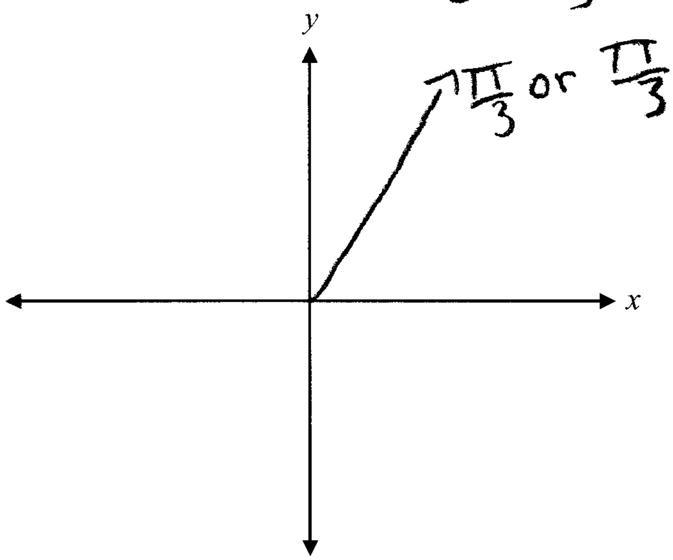


½ out of 1

+ ½ mark for correct number of revolutions

Exemplar 3

$$\frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$$



½ out of 1

+ ½ mark for an appropriate angle in quadrant I

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Question 7 R11

Determine, algebraically, all of the zeros of the polynomial function $P(x) = x^4 - 5x^3 - 4x^2 + 20x$.

Solution

$P(x) = x\left(x^3 - 5x^2 - 4x + 20\right)$

$P(2) = 2\left(2^3 - 5(2)^2 - 4(2) + 20\right)$ 1 mark for identifying one possible value of x

$P(2) = 0$

$\therefore (x-2)$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -5 & -4 & 20 \\ & \downarrow & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

1 mark for synthetic division
(or any other equivalent strategy)

$P(x) = x(x-2)\left(x^2 - 3x - 10\right)$

$0 = x(x-2)(x-5)(x+2)$ ½ mark for identifying all the factors

$x = 0,\ x = 2,\ x = 5,\ x = -2$ ½ mark for consistent zeros

3 marks

Exemplar 1

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -4 & 20 \\ & & -2 & 14 & -20 \\ \hline & 1 & -7 & 10 & 0 \end{array}$$

$x^3 - 7x^2 + 10x$

$x(x^3 - 7x + 10)$

$(x-2)(x-5)$

Zero's $= x, (x+2), (x-2), (x-5)$

2 out of 3

+ 1 mark for identifying one possible value of x
+ 1 mark for synthetic division

Exemplar 2

$$\begin{array}{c|cccc} 2 & 1 & -5 & -4 & 20 \\ & & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$$\boxed{\text{Zeros: } 2, 5, -2}$$

$$(x-2)(x^2-3x-10)$$

$$(x-2)(x-5)(x+2)$$

2 out of 3

+ 1 mark for identifying one possible value of x
+ 1 mark for synthetic division
+ ½ mark for consistent zeros
– ½ mark for procedural error in line 3

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Question 8 P4

Justify why four of the terms in the binomial expansion of $(-x+y)^6$ are positive.

Solution

$(-x)^6(y)^0, (-x)^5(y)^1, (-x)^4(y)^2, \ldots$

$x^6, -x^5y, x^4y^2, \ldots$

There are 7 terms. The first term is positive and the signs alternate.

1 mark for justification

1 mark

Exemplar 1

because every even k value
will give you a positive
term and there is 4 even
numbers between (0-6) inclusive.

1 out of 1

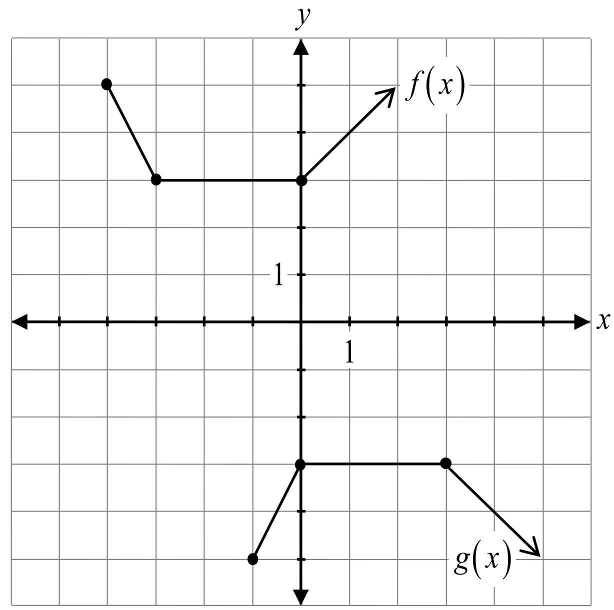
Exemplar 2

In every even term the negative α would become positive due to the exponent.
In the last term the negative α would be to the power of 0, removing the negative
again.

½ out of 1

award full marks
– ½ mark for terminology error

Determine the equation of $g(x)$ in terms of $f(x)$.



Solution

$g(x) = \underline{-f(x-3)}$ 1 mark for vertical reflection
1 mark for horizontal translation

2 marks

Exemplar 1

$g(x) =$ $-1(x-3)$

1 out of 2

award full marks

– 1 mark for concept error (not including $f(x)$)

Exemplar 2

$g(x) =$ $-1(f(x-3))-6$

1 out of 2

award full marks

– 1 mark for concept error (reflection over $y = 3$)

Question 10 T6

Prove the identity for all permissible values of x.

$$\frac{\sin x + \tan x}{\cot x + \csc x} = \frac{\sin^2 x}{\cos x}$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{\sin x + \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{1}{\sin x}}$	$\dfrac{\sin^2 x}{\cos x}$
$\dfrac{\dfrac{\sin x \cos x + \sin x}{\cos x}}{\dfrac{\cos x + 1}{\sin x}}$	
$\dfrac{\sin x \cancel{\left(\cos x + 1\right)}}{\cos x} \bullet \dfrac{\sin x}{\cancel{\left(\cos x + 1\right)}}$	
$\dfrac{\sin^2 x}{\cos x}$	

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Solution

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{\sin x + \tan x}{\dfrac{1}{\tan x} + \dfrac{1}{\sin x}}$	$\dfrac{\sin^2 x}{\cos x}$
$\dfrac{\sin x + \tan x}{\dfrac{\sin x + \tan x}{\sin x \tan x}}$	
$\dfrac{\cancel{\sin x + \tan x}}{\cancel{\sin x + \tan x}} \bullet \sin x \tan x$	
$\sin x \bullet \dfrac{\sin x}{\cos x}$	
$\dfrac{\sin^2 x}{\cos x}$	

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\dfrac{\left(\frac{\cos}{\cos}\right)\frac{\sin}{1} x + \frac{\sin}{\cos}(}{\frac{\cos}{\sin} + \frac{1}{\sin}}$	$\dfrac{\sin^2 x}{\cos x}$
$\dfrac{\frac{\cancel{\cos x}\sin x}{\cancel{\cos}}}{\frac{\cos x}{\sin x}}$	
$\dfrac{\sin x}{\frac{\cos}{\sin}}$	
$\dfrac{\sin x}{1} \cdot \dfrac{\sin x}{\cos x}$	

$$= \frac{\sin^2}{\cos^x}$$ ✓

1 out of 3

+ 1 mark for correct substitution of identities
E3 (variable omitted in an identity)

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{\sin x + \tan x}{\dfrac{\cos x}{\sin x} + \dfrac{1}{\sin x}}$	$\dfrac{\sin^2 x}{\cos x}$

0 out of 3

Exemplar 3

Left-Hand Side	Right-Hand Side
$LHS = \dfrac{\sin x + \tan x}{\cot x + \csc x}$	$RHS = \dfrac{\sin^2 x}{\cos x}$
$(\cos x)\dfrac{\dfrac{\sin x}{1} + \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{1}{\sin x}}$	
$\dfrac{\dfrac{\cos x \sin x + \sin x}{\cos x}}{\dfrac{1 + \cos x}{\sin x}}$	

2 out of 3

+ 1 mark for correct substitution of identities
+ 1 mark for algebraic strategies

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Question 11 R3

Given that the point $(-2,1)$ is on the graph of $y = f(x)$, describe how the coordinates of the corresponding point on the graph of $y = f(4x)$ are different.

Solution

The x-value is divided by 4.

1 mark

Exemplar 1

$y=1$ will stay the same because there are no changes for y.

x will be compressed by 4 so $x=-2$ will be $x=-\frac{1}{2}$.

1 out of 1

Exemplar 2

$(-2,1) \longrightarrow \boxed{\left(\frac{-1}{2},1\right)}$

0 out of 1

Exemplar 3

The x value is different.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 12 R8

Using the laws of logarithms, completely expand the expression:

$$\log\left(\frac{x^2\sqrt{y}}{w-1}\right)$$

Solution

$$2\log x + \frac{1}{2}\log y - \log(w-1)$$

1 mark for product law
1 mark for power law (½ mark for each)
1 mark for quotient law

3 marks

Exemplar 1

$$= \log x^2 + \log y^{0.5} - \left(\frac{\log w}{\log 1}\right)$$

$$= \log x^2 + 0.5\log y - \left(\frac{\log w}{\log 1}\right)$$

1½ out of 3

+ 1 mark for product law
+ ½ mark for power law

Exemplar 2

$$\frac{\log x^2 + \log\sqrt{y}}{\log(w-1)}$$

$$2\log x + \frac{1}{2}\log y - \log(w-1)$$

$$\boxed{2\log x + \frac{1}{2}\log y - \log w - \log 1}$$

2 out of 3

+ 1 mark for product law
+ 1 mark for power law

Exemplar 3

$$\log x^2 + \log\sqrt{y} - (\log w - 1)$$

$$2\log x + \log y^{\frac{1}{2}} - (\log w - 1)$$

$$2\log x + \frac{1}{2}\log y - (\log w - 1)$$

3 out of 3

award full marks
E7 (notation error in lines 1 to 3)

Question 13 T3

Given $\sec\theta = -\frac{5}{4}$ and $\tan\theta > 0$, state the quadrant in which θ terminates.

Justify your answer.

Solution

Since $\sec\theta$ is negative in quadrants II and III, and $\tan\theta$ is positive in quadrants I and III, θ terminates in quadrant III.

1 mark for justification

1 mark

Exemplar 1

$\cos\theta = -\frac{4}{5}$ $\tan\theta > 0$

[Quadrant III]

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 2

For tan to be greater than 0 it must be Q1 or Q3.
For secant to be negative it must be Q3 or Q2.
∴ θ terminates in Q2.

1 out of 1

award full marks
E7 (transcription error)

Exemplar 3

III

0 out of 1

Question 14 R14

State an equation of a rational function that has a vertical asymptote at $x = -8$ and a horizontal asymptote at $y = 9$.

Solution

$$y = \frac{9x}{x+8}$$

1 mark for vertical asymptote
1 mark for horizontal asymptote

2 marks

Note:

- Other equations are possible.

Exemplar 1

$$f(x) = \frac{1}{(x+8)} + 9$$

2 out of 2

Exemplar 2

$$\frac{9(x)}{(x+8)}$$

2 out of 2

award full marks
E2 (changing an equation to an expression)

Exemplar 3

$$y = \frac{(x+8)(x-9)}{(x+8)}$$

0 out of 2

Exemplar 4

$$y = \frac{9x^2+2}{(x^2-64)}$$

2 out of 2

Question 15 R5

Given the point $(5, 1)$, state the coordinates of the corresponding point after a reflection across the line $y = x$.

Solution

$(1, 5)$ **1 mark**

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Simplify.

$$\frac{(n-13)!}{(n-12)!}$$

Solution

$$\frac{\cancel{(n-13)!}}{(n-12)\cancel{(n-13)!}}$$

1 mark for factorial expansion

$$\frac{1}{n-12}$$

1 mark

Exemplar 1

$$= \frac{(n-\cancel{13})(n-\cancel{14})}{(n-12)(n-\cancel{13})(n-\cancel{14})}$$

$$= \frac{1}{(n-12)}$$

1 out of 1

award full marks
E7 (notation error)

Exemplar 2

$$\frac{\cancel{(n-13)!}}{(n-12)\cancel{(n-13)!}}$$

$$n-12$$

½ out of 1

award full marks
– ½ mark for arithmetic error

Exemplar 3

$$\frac{(n-13)!}{(n-12)(n-13)!}$$

1 out of 1

E1 (final answer not stated)

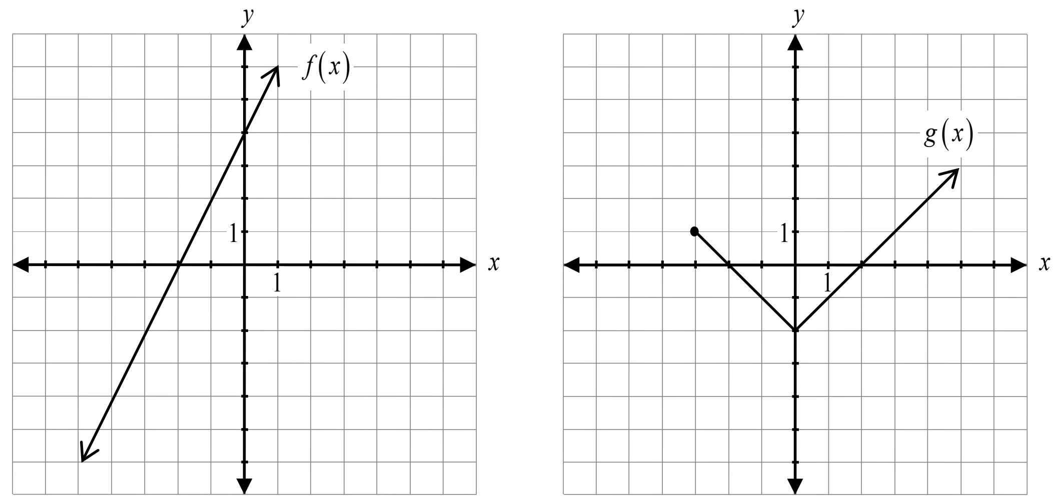
Exemplar 4

$$\frac{(n-\cancel{13})!}{(n-12)(n-\cancel{13})!} \qquad \frac{1}{(n-12)!}$$

½ out of 1

award full marks
– ½ mark for procedural error

Given the graphs of $y = f(x)$ and $y = g(x)$,



Solution

a) determine the value of $f(g(2))$.

$g(2) = 0$ ½ mark for the value of $g(2)$

$f(0) = 4$ ½ mark for the value of $f(g(2))$ consistent with $g(2)$

1 mark

b) determine the value of $(g - f)(-3)$.

$g(-3) - f(-3)$

$1 - (-2)$ ½ mark for the values of $g(-3)$ and $f(-3)$

3 ½ mark for value of $(g - f)(-3)$ consistent with $g(-3)$ and $f(-3)$

1 mark

Exemplar 1

a) $g(2) = 1$

$f(1) = 6$

½ out of 1

+ ½ mark for the value of $f(g(2))$ consistent with $g(2)$

b) $f(-3) = -2$

$g(-3) = 1$

½ out of 1

+ ½ mark for the values of $g(-3)$ and $f(-3)$

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
18	D	R11
19	C	R7
20	A	P3
21	B	R1
22	C	T6
23	A	P4
24	A	R13
25	D	T1
26	C	R1
27	A	R2

Question 18 R11

Identify the remainder when $P(x) = 3x^3 - x^2 + 1$ is divided by $(x - 2)$.

a) -27

b) -19

c) 11

d) 21

Question 19 R7

Identify the logarithmic form of $2^x = \frac{1}{4}$.

a) $\log_2 x = \frac{1}{4}$

b) $\log_x 2 = \frac{1}{4}$

c) $\log_2 \left(\frac{1}{4}\right) = x$

d) $\log_x \left(\frac{1}{4}\right) = 2$

Question 20 P3

Leah's Pizzeria offers 9 different pizza toppings. Identify the expression that represents the number of different types of pizzas, with 3 different toppings, that can be made.

a) ${}_9C_3$

b) ${}_9P_3$

c) $\frac{9!}{3!}$

d) $9!3!$

Question 21 R1

Given $(5,-4)$ is a point on the graph of $y = f(x)$, identify the corresponding point on the graph of $y = \frac{1}{f(x)}$.

a) $\left(\frac{1}{5}, -4\right)$

b) $\left(5, -\frac{1}{4}\right)$

c) $\left(\frac{1}{5}, -\frac{1}{4}\right)$

d) $(-4, 5)$

Question 22 T6

Identify the non-permissible value of x for $1 + \sec x$ over $[0, \pi]$.

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) π

Question 23 P4

Indicate the combination that represents the circled term in the given row of Pascal's triangle.

1 4 6 4 1

a) ${}_4C_3$

b) ${}_4C_4$

c) ${}_5C_3$

d) ${}_5C_4$

Question 24 R13

Identify the x-intercept on the graph of $f(x) = \sqrt{2(x+5)}$.

a) -5

b) 0

c) $\sqrt{10}$

d) 5

Question 25 T1

Identify the coterminal angle of $\frac{\pi}{5}$ over the interval $-\pi \le \theta \le 4\pi$.

a) $-\frac{9\pi}{5}$

b) $-\frac{\pi}{5}$

c) $\frac{3\pi}{5}$

d) $\frac{11\pi}{5}$

Question 26 R1

Given $f(x) = \{(2,6),(3,2),(3,4),(6,5)\}$, identify the value of $f(f(2))$.

a) 3

b) 4

c) 5

d) 6

Question 27 R2

The graph of $f(x)=(x-1)^2$ is translated 2 units to the left and 3 units up. Identify the equation of the transformed graph, $g(x)$.

a) $g(x)=(x+1)^2+3$

b) $g(x)=(x-3)^2+3$

c) $g(x)=(x+2)^2+3$

d) $g(x)=(x-2)^2+3$

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Question 28 T6

Given $\csc\theta = -\frac{8}{5}$, determine the exact value of $\cos 2\theta$.

Solution

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$= 1 - 2\left(-\frac{5}{8}\right)^2$$

1 mark for the value of $\sin\theta$
1 mark for substitution into correct identity

$$= \frac{14}{64}$$

2 marks

or

$$= \frac{7}{32}$$

Exemplar 1

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(-\frac{5}{8}\right)^2$$
$$= 1 - 2\left(\frac{25}{64}\right)$$
$$= 1 - \frac{50}{64}$$
$$\boxed{\cos 2\theta = 14}$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 5
E1 (impossible solution not rejected in final answer)

Exemplar 2

$$\cos 2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(-\frac{5}{8}\right)^2$$
$$= 1 - \frac{25}{64}$$
$$= \frac{64}{64} - \frac{25}{64}$$
$$\boxed{\cos 2\theta = \frac{39}{64}}$$

1 out of 2

+ 1 mark for the value of $\sin\theta$

Exemplar 3

$$\cos 2\theta = 1 - 2\left(\frac{5}{8}\right)^2$$
$$= 1 - 2\left(\frac{25}{64}\right)$$
$$\frac{64}{64} - \frac{50}{64}$$
$$\frac{14}{64}$$

1½ out of 2

award full marks

– ½ mark for procedural error in line 1

E2 (changing an equation to an expression)

Exemplar 4

$$\cos\theta = \frac{-5}{8}$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(\frac{-5}{8}\right)^2 - 1$$
$$= 2\left(\frac{25}{64}\right) - 1$$
$$= \frac{50}{64} - \frac{64}{64}$$
$$= \frac{-14}{64}$$

1 out of 2

+ 1 mark for substitution into correct identity

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Question 29 T4

Determine the period of the sinusoidal function, $f(x) = -6\cos\left(\frac{\pi}{6}(x+1)\right) + 5.$

Solution

$$\text{Period} = \frac{2\pi}{\left(\frac{\pi}{6}\right)}$$

$$= 12$$

1 mark

Exemplar 1

period $= \dfrac{\pi}{\frac{\pi}{6}}$

$\cancel{\pi} \cdot \dfrac{6}{\cancel{\pi}}$

$= 6$

0 out of 1

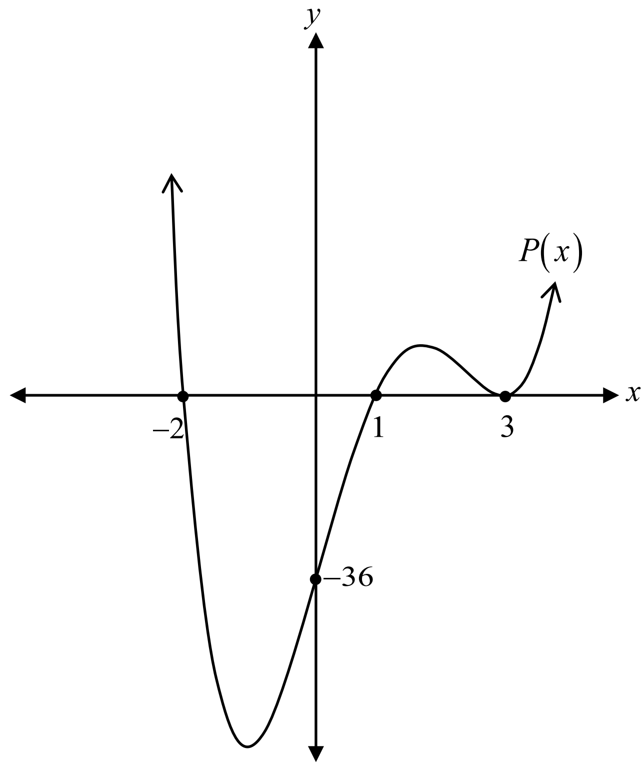
Exemplar 2

Period $= \dfrac{2\pi}{\frac{\pi}{6}}$

1 out of 1

award full marks
E1 (final answer not stated)

Determine, algebraically, the equation of $P(x)$, given the graph of the polynomial function $P(x)$.



Solution

$P(x) = a(x-3)^2(x-1)(x+2)$ — ½ mark for factors of $P(x)$; ½ mark for multiplicity of 2 at $x = 3$

$-36 = a(-3)^2(-1)(2)$ — ½ mark for substitution of $P(0) = -36$

$-36 = -18a$

$a = 2$ — ½ mark for correct value of a

$P(x) = \underline{2(x-3)^2(x-1)(x+2)}$

2 marks

Exemplar 1

$(x+2)(x-1)\ (x-3)(x-3)$

$(x^2+x-2)(x^2-6x+9)$

$x^4-6x^3+9x^2+x^3-6x^2+9x-2x^2+12x-18$

$2(x^4-5x^3+x^2+21x-18)$

$P(x) = 2x^4-10x^3+2x^2+42x-36$

2 out of 2

Exemplar 2

$P(x) = (x+2)(x-1)(x-3)^2-36$

1 out of 2

+ ½ mark for factors of $P(x)$

+ ½ mark for multiplicity of 2 at $x = 3$

Question 31 T5

Solve $2\sin^2\theta - 7\sin\theta - 4 = 0$ where $\theta \in \mathbb{R}$.

Solution

$(2\sin\theta + 1)(\sin\theta - 4) = 0$

$\sin\theta = -\frac{1}{2}$	$\sin\theta = 4$	1 mark for solving for $\sin\theta$ (½ mark for each branch)
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$	No solution	2 marks for solving for θ (1 mark for each branch)
$\theta = \frac{7\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$		
$\theta = \frac{11\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$		1 mark for general solution

4 marks

Exemplar 1

$2x^2 - 2x - 4 = 0$

$\sin\theta = 4$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}$

$\theta = 2\pi \cdot \frac{7\pi}{6} = \frac{5\pi}{6}$

$\frac{7\pi}{6} + 2\pi k, k \in R$

$\frac{5\pi}{6} + 2\pi k, k \in R$

2 out of 4

+ 1 mark for solving for $\sin\theta$
+ ½ mark for solving for θ
+ 1 mark for general solution
– ½ mark for procedural error in lines 6 and 7 ($k \in \mathbb{R}$)
E3 (variable introduced without being defined)
E2 (changing an equation to an expression)

Exemplar 2

$(2\sin\theta+4)(\sin\theta-1)=0$

$2\sin\theta+4=0$

$\sin\theta=\frac{-4}{2}=-2$

no solution

$\sin\theta-1=0$

$\sin\theta=1$

$\theta=\frac{\pi}{2}+2\pi k, k\in Z$

$\frac{3\pi}{2}+2\pi k, k\in Z$

2½ out of 4

award full marks
– ½ mark for arithmetic error in line 1
– 1 mark for concept error in line 5

Exemplar 3

$(2\sin\theta+1)(\sin\theta-4)=0$

$\sin\theta=-1/2$ $\quad$ $\cancel{\sin\theta=4}$

$\theta ref=30°$

$\theta=210°$

$\theta=330°$

3 out of 4

+ 1 mark for solving for $\sin\theta$
+ 2 marks for solving for θ

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Question 32 R12

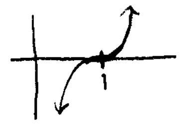
Justify that the shapes of the graphs of $f(x) = (x+1)^2 (x-1)$ and $g(x) = (x+1)^2 (x-1)^3$ are different as they approach the x-intercept at $x = 1$.

Solution

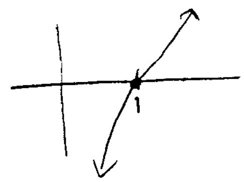
At $x = 1$, both graphs pass through the x-axis, however, the graph of $g(x)$ flattens as it passes through the x-axis.

1 mark

Exemplar 1

The $g(x)$ graph will have a 3 multiplicitcy that will look like this 

as the $f(x)$ graph will go straight through.



1 out of 1

Exemplar 2

The graph will have a different type of curve going through the coordinate of $(1, 0)$.

0 out of 1

Exemplar 3

The graph of $f(x)$ would pass through the graph at $x=1$, the graph $g(x)$ would "slide" along the x-axis before passing through.

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 4

The graph of $g(x)$ will flatten as it crosses the zero at $x=1$, while $f(x)$ will not flatten.

1 out of 1

Question 33 T2

Determine the exact value of $\cot\theta$ if $\cos\theta = -\frac{4}{7}$ and $\sin\theta$ is positive.

Solution

$x^2 + y^2 = r^2$

$(-4)^2 + y^2 = (7)^2$ ½ mark for substitution

$y^2 = 49 - 16$

$y = \pm\sqrt{33}$ ½ mark for solving for y

$\cot\theta = \frac{-4}{\sqrt{33}}$ 1 mark for consistent value of $\cot\theta$ (½ mark for quadrant; ½ mark for value)

or

$\cot\theta = \frac{-4\sqrt{33}}{33}$

2 marks

Note:

- Accept any of the following values for y : $y = \pm\sqrt{33}$, $y = \sqrt{33}$, or $y = -\sqrt{33}$.

Exemplar 1

$$7^2 - 4^2 = b^2$$

$$41 - 16 = b^2$$

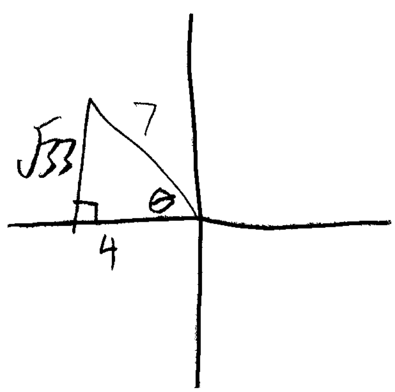
$$\sqrt{35} = b$$

$$\cot\theta = \frac{4}{\sqrt{35}}$$

1 out of 2

+ ½ mark for substitution
+ ½ mark for consistent value of $\cot\theta$

Exemplar 2



$$4^2 + y^2 = 7^2$$

$$16 + y^2 = 49$$

$$y^2 = 33$$

$$y = \sqrt{33}$$

$$\tan\theta = \frac{\sqrt{33}}{7}$$

$$\cot\theta = \frac{7}{\sqrt{33}}$$

1 out of 2

+ ½ mark for substitution
+ ½ mark for solving for y

Exemplar 3

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\frac{-4}{7}}{\sqrt{33}}$$

$$= \frac{-4}{7} \times \frac{1}{\sqrt{33}}$$

$$\boxed{\cot\theta = \frac{-4}{7\sqrt{33}}}$$

1 out of 2

award full marks
– 1 mark for concept error in line 2

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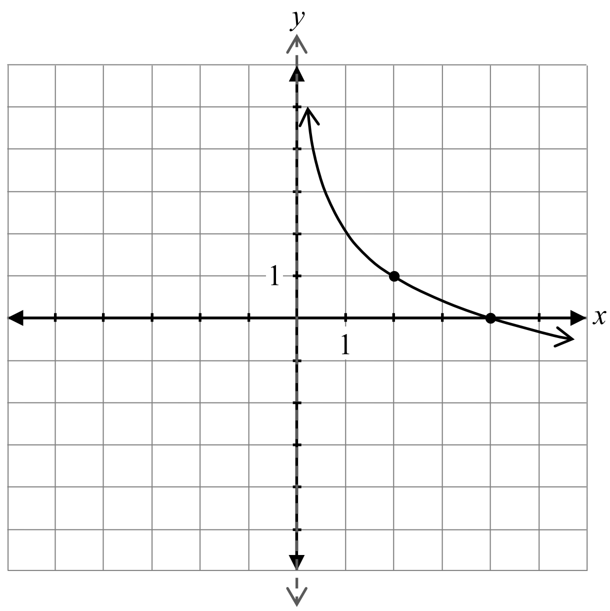
Sketch the graph of $f(x) = -\log_2(x) + 2$.

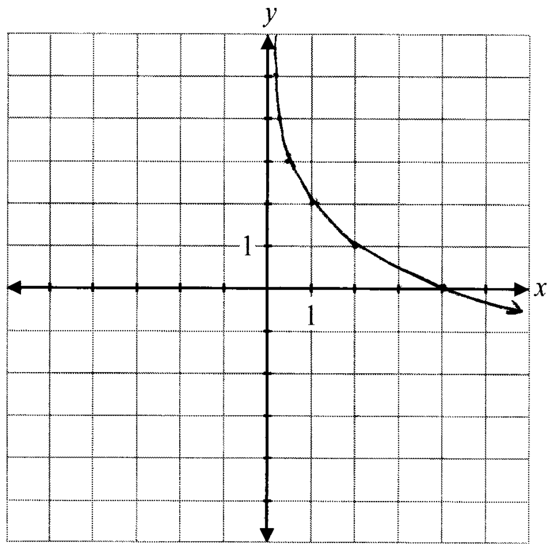
Solution



1 mark for asymptotic behaviour approaching $x = 0$
1 mark for vertical reflection
1 mark for vertical translation

3 marks

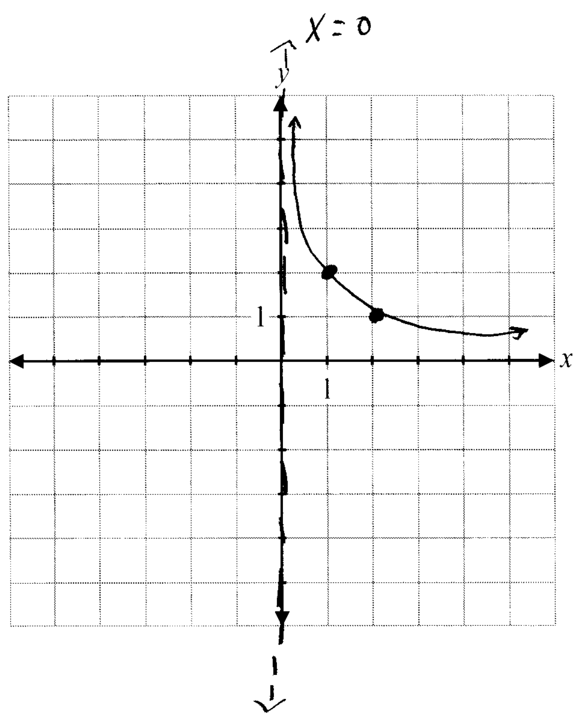
Exemplar 1



3 out of 3

award full marks
E10 (asymptote omitted but still implied)
E9 (arrowhead omitted)

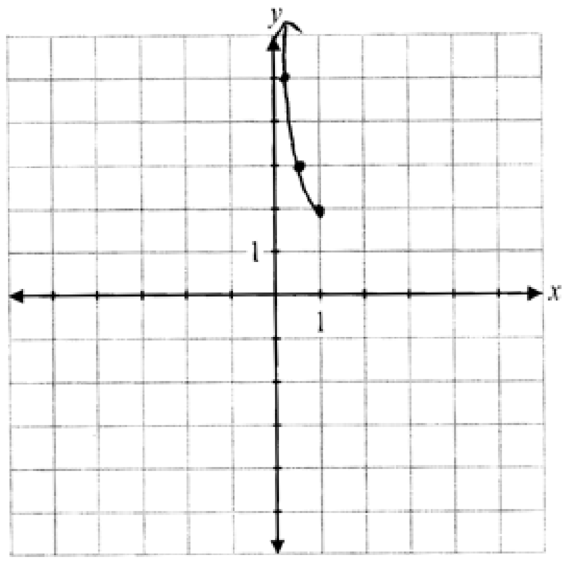
Exemplar 2



2½ out of 3

award full marks
– ½ mark for procedural error (x-intercept omitted)

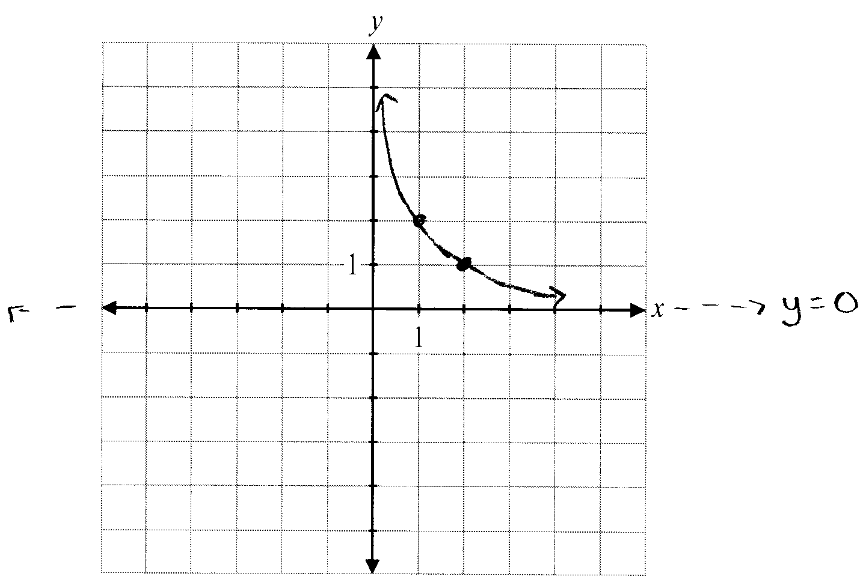
Exemplar 3



2 out of 3

award full marks
– ½ mark for procedural error (x-intercept omitted)
– ½ mark for incorrect shape
E10 (asymptote omitted but still implied)

Exemplar 4



2 out of 3

award full marks
– 1 mark for concept error (including a horizontal asymptote)

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Question 35 R13

State the range of $f(x) = \sqrt{x+4}$.

Solution

Range: $[0, \infty)$ **1 mark**

or

Range: $\{y \in \mathbb{R} \mid y \geq 0\}$

Exemplar 1

Range: $(0, \infty)$

1 out of 1

award full marks
E8 (bracket error made when stating domain)

Exemplar 2

Range: $(\infty, 0]$

1 out of 1

award full marks
E8 (range written in incorrect order)

Exemplar 3

Range: $y \geq 0$

1 out of 1

Question 36 R10

Sophie correctly solved the logarithmic equation, $\log_7(x-1) = \log_7(2x-2)$.

$$x - 1 = 2x - 2$$

$$-1 + 2 = 2x - x$$

$$\cancel{1 = x}$$

Explain why $x = 1$ is an extraneous root.

Solution

The root, $x = 1$, is an extraneous root because the argument of a logarithm cannot be zero.

1 mark

Exemplar 1

If you substitute the answer you get back into the equation and it doesnt work, it is extraneous.

0 out of 1

Exemplar 2

You can't have a negative argument in a logarithmic equation so if you solve & get a value that's a negative, it is an extraneous root.

0 out of 1

Exemplar 3

you can't take the log of zero.

1 out of 1

Exemplar 4

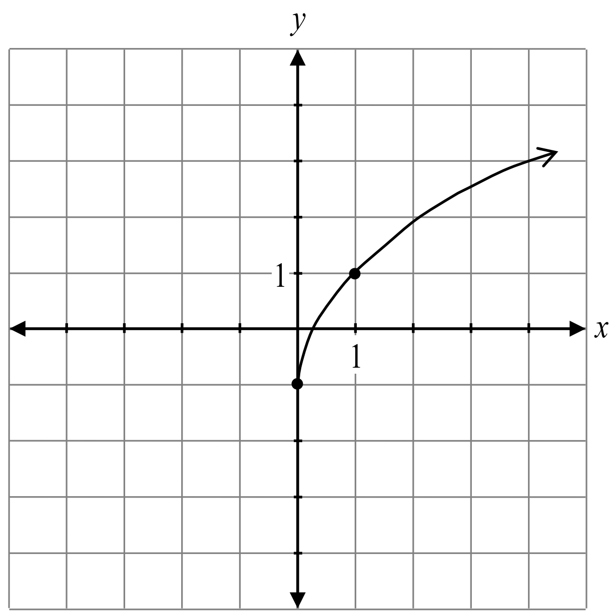
When you plug x=1 into the equation, you get 0.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Sketch the graph of $f(x) = \sqrt{4x} - 1$.

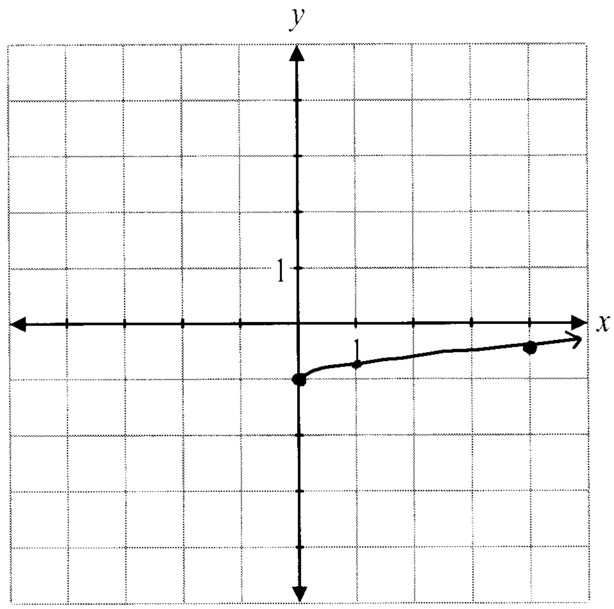
Solution



1 mark for shape of a radical function
1 mark for horizontal compression
1 mark for vertical translation

3 marks

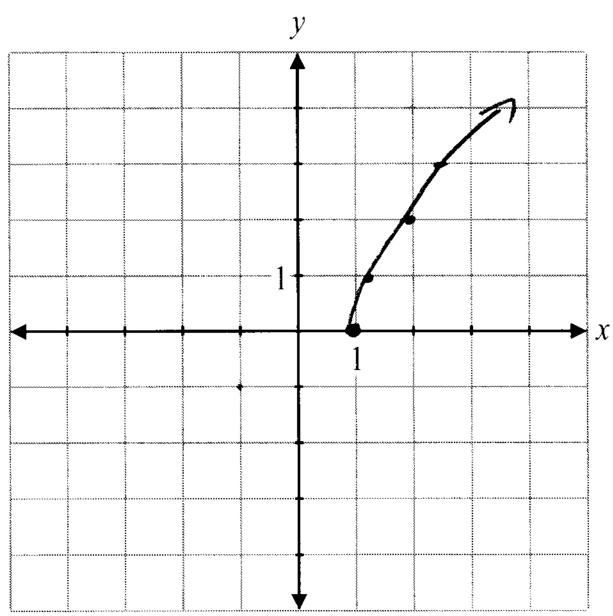
Exemplar 1



2 out of 3

+ 1 mark for shape of a radical function
+ 1 mark for vertical translation

Exemplar 2



2 out of 3

+ 1 mark for shape of a radical function
+ 1 mark for horizontal compression

Question 38 P3

Solve, algebraically.

$${}_nC_2 = 2n + 7$$

Solution

$$\frac{n!}{(n-2)!2!} = 2n + 7$$ ½ mark for substitution into equation

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 2!(2n+7)$$ ½ mark for factorial expansion
½ mark for simplification of factorial

$$n(n-1) = 2(2n+7)$$

$$n^2 - n = 4n + 14$$

$$n^2 - 5n - 14 = 0$$ ½ mark for simplification

$$(n+2)(n-7) = 0$$

$$\cancel{n = -2} \qquad n = 7$$ ½ mark for the permissible value of n
½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\frac{n!}{(n-2)!} = 2n+7$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)}!} = 2n+7$$

$$n(n-1) = 2n+7$$

$$\begin{aligned} n^2 - n &= 2n+7 \\ -2n-7 \quad & \quad -2n-7 \end{aligned}$$

$$n^2 - 3n - 7 = 0$$

1½ out of 3

+ ½ mark for factorial expansion
+ ½ mark for simplification of factorial
+ ½ mark for simplification

Exemplar 2

$$\frac{n!}{n-2!\,2!} = 2n+7$$

$$\frac{(n)(n-1)\cancel{(n-2)!}}{\cancel{n-2!}\;2!} = 2n+7$$

$$\frac{n^2-n}{2!} = 2n+7$$

$$n^2-n = 4n+14$$

$$n^2-5n-14=0$$

$$(n-7)(n+2)=0$$

$$n=7 \quad \cancel{n=2}$$

$$n=7$$

3 out of 3

award full marks
E4 (missing brackets but still implied in lines 1 and 2)

Exemplar 3

$$\frac{n!}{2!(n-2!)} = 2n+7$$

$$\frac{n(n-1)}{2} = 2n+7$$

$$n^2 - n = 4n + 14$$

$$n^2 - 5n - 14 = 0$$

$$(n-7)(n+2) = 0$$

$$n = +7$$

2 ½ out of 3

+ ½ mark for substitution into equation
+ ½ mark for factorial expansion
+ ½ mark for simplification of factorial
+ ½ mark for simplification
+ ½ mark for the permissible value of *n*
E7 (notation error in line 1)

Question 39 R1

Given $f(x) = x^2 - 1$ and $g(x) = x - 3,$ explain why the domain of $h(x) = \dfrac{f(x)}{g(x)}$ has a restriction when $x = 3.$

Solution

When $x = 3,$ the denominator is equal to zero and it is not possible to divide by zero.

1 mark

Exemplar 1

$$\frac{x^2-1}{x-3}$$

* It has a restriction of 3 because that's where the graph does not exists because $x=3$ is an asymptote.

1 out of 1

Exemplar 2

because the domain can only be where both graphs exist.

0 out of 1

Exemplar 3

when $x=3$ $hx=\frac{9}{0}$ and because we can't divide by 0, the point at $x=3$ becomes a point of discontinuity

0 out of 1

Exemplar 4

$h(x)=\frac{3^2-1}{3-3}$ $h(x)=\frac{8}{0}$ = undefined

$h(x)$ is undefined when $x=3$

0 out of 1

Exemplar 5

$h(x)=\frac{x^2-1}{x-3}$

NPV is $x \neq 3$

because it's on the denominator

0 out of 1

Question 40 T3

Evaluate.

$$\frac{\cot\left(\frac{11\pi}{6}\right)\sin\left(-\frac{4\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}$$

Solution

$$\frac{\left(-\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}}$$

$$\left(-\frac{3}{2}\right)\left(-\frac{2}{1}\right)$$

$$3$$

1 mark for $\cot\left(\frac{11\pi}{6}\right)$ (½ mark for quadrant; ½ mark for value)

1 mark for $\sin\left(-\frac{4\pi}{3}\right)$ (½ mark for quadrant; ½ mark for value)

1 mark for $\cos\left(\frac{2\pi}{3}\right)$ (½ mark for quadrant; ½ mark for value)

3 marks

Exemplar 1

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\frac{\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right)}{-\frac{\sqrt{3}}{2}}$$

$$\frac{\frac{1}{2\sqrt{3}}}{\frac{\sqrt{3}}{2}} \qquad \frac{1}{2\sqrt{3}} \times \frac{2}{\sqrt{3}} \qquad = \frac{2}{2(3)}$$

$$= \frac{2}{6}$$

$$\boxed{= \frac{1}{3}}$$

1 out of 3

+ ½ mark for quadrant of $\sin\left(-\frac{4\pi}{3}\right)$

+ ½ mark for quadrant of $\cos\left(\frac{2\pi}{3}\right)$

E7 (transcription error in line 2)

Exemplar 2

$$\frac{\left(\frac{\sqrt{3}}{1}\right)\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$$

$$\frac{-3}{\cancel{2}}\left(\frac{\cancel{2}}{1}\right)$$

$$\boxed{-3}$$

1½ out of 3

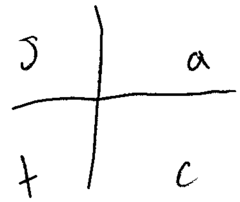
+ ½ mark for value of $\cot\left(\frac{11\pi}{6}\right)$

+ ½ mark for value of $\sin\left(-\frac{4\pi}{3}\right)$

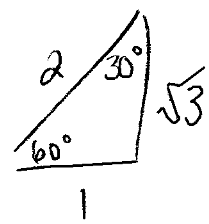
+ 1 mark for value of $\cos\left(\frac{2\pi}{3}\right)$

– ½ mark for arithmetic error in line 2

Exemplar 3



$$\frac{\frac{\cos}{\sin}\left(\frac{11\pi}{6}\right)\sin\left(-\frac{4\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}$$



$$\frac{\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}}$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right)\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right)$$

$$\left(\frac{3}{4}\right)\left(-\frac{4}{1}\right)$$

$$\frac{-12}{4}$$

$$-3$$

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 4
E7 (notation error in line 1)

Question 41 R10

Solve, algebraically.

$$\log_2\left(\log_3 x\right) = 2$$

Solution

$2^2 = \log_3 x$ ½ mark for exponential form

$4 = \log_3 x$

$x = 3^4$ ½ mark for exponential form

$x = 81$

1 mark

Exemplar 1

$2^2 = x$

$4 = x$

0 out of 1

Exemplar 2

$\log_2 2 = \log_3 x$

$1 = \log_3 x$

$3^1 = x$

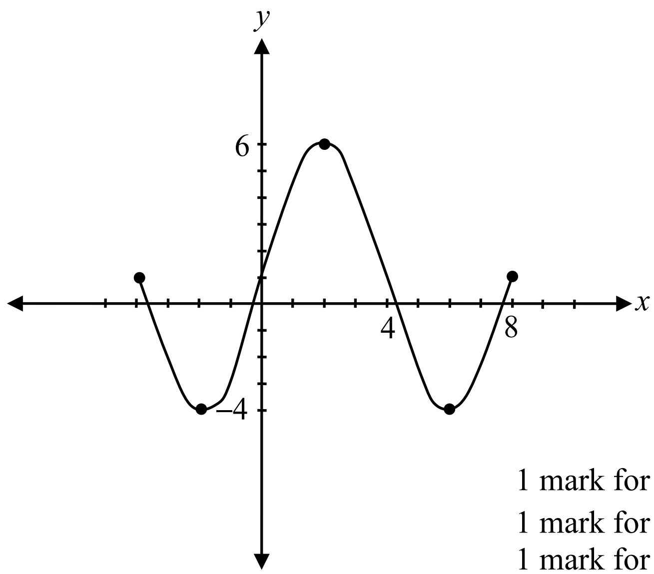
$3 = x$

½ out of 1

+ ½ mark for exponential form

Sketch the graph of the function $y = 5\sin\left(\frac{\pi}{4}x\right)+1$ over the domain $[-4, 8]$.

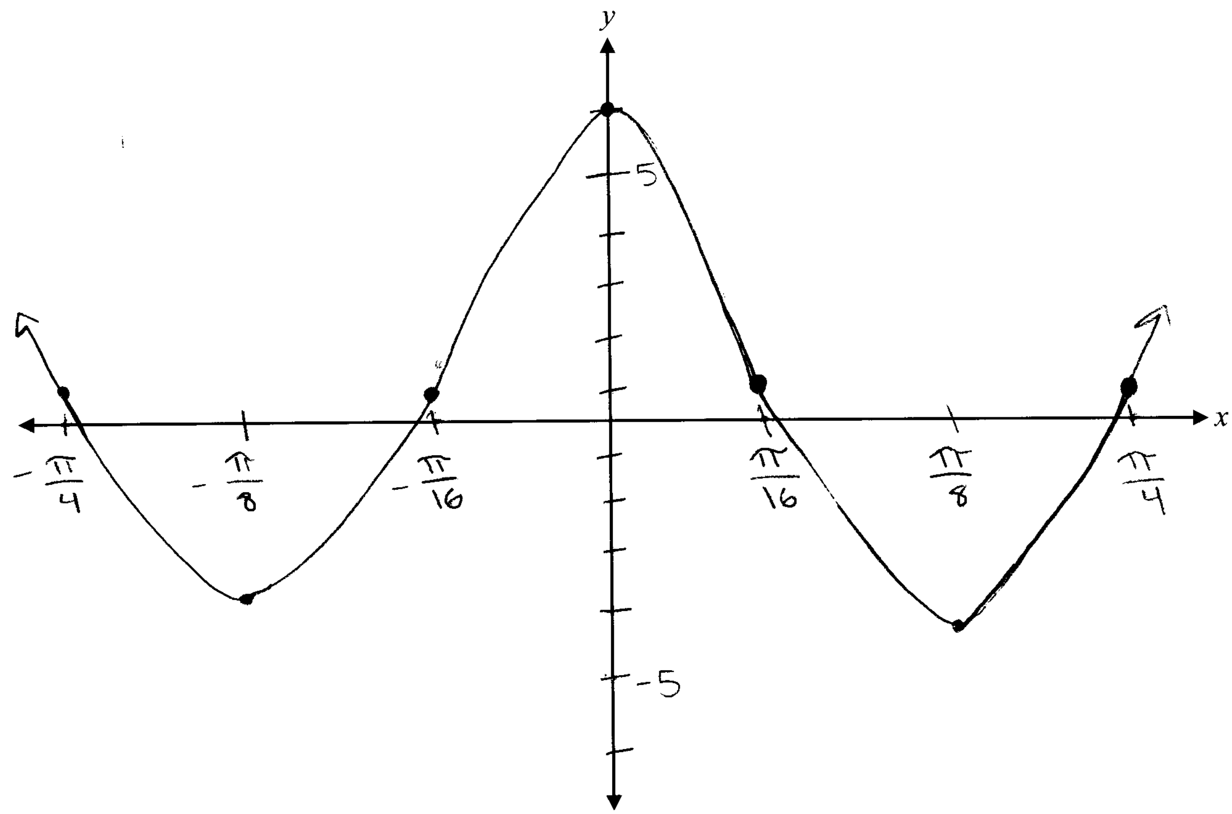
Solution



1 mark for shape of $y = \sin x$
1 mark for amplitude
1 mark for period
1 mark for vertical translation

4 marks

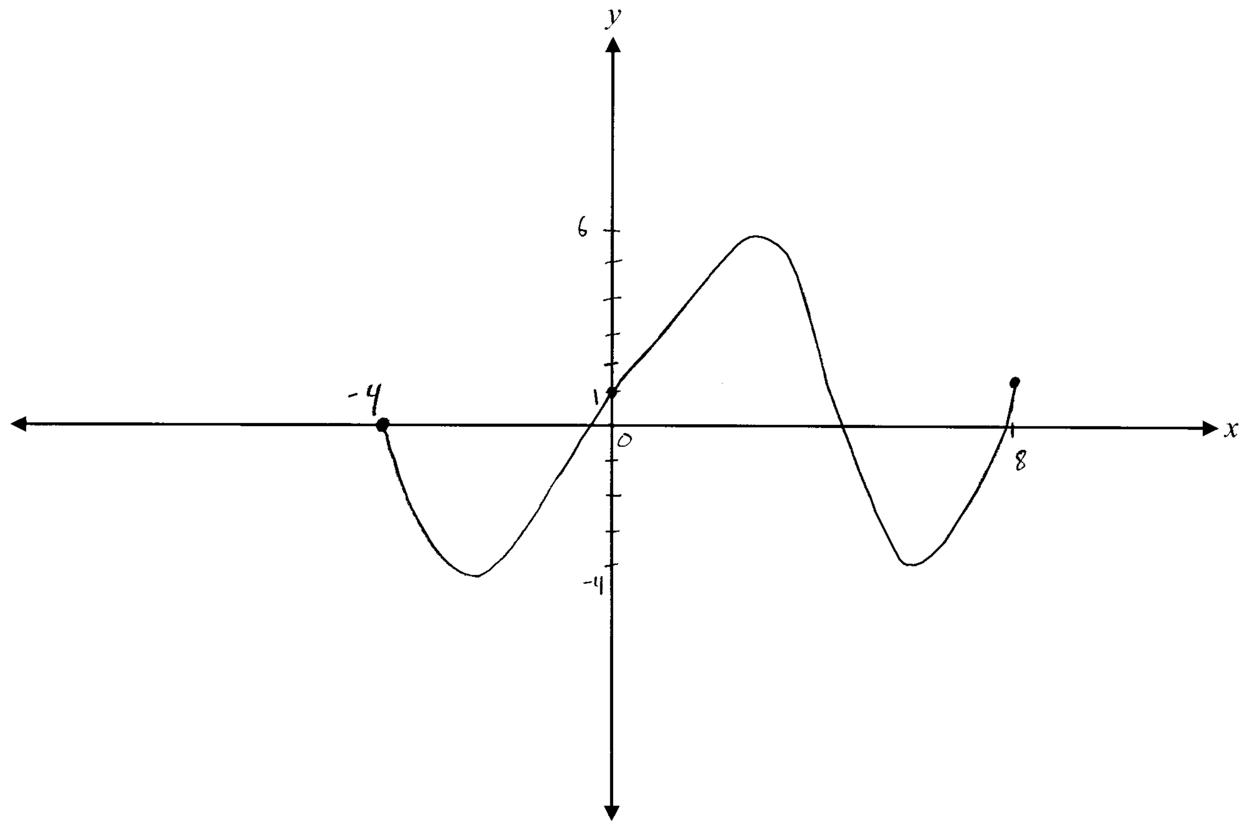
Exemplar 1



2 out of 4

+ 1 mark for amplitude
+ 1 mark for vertical translation

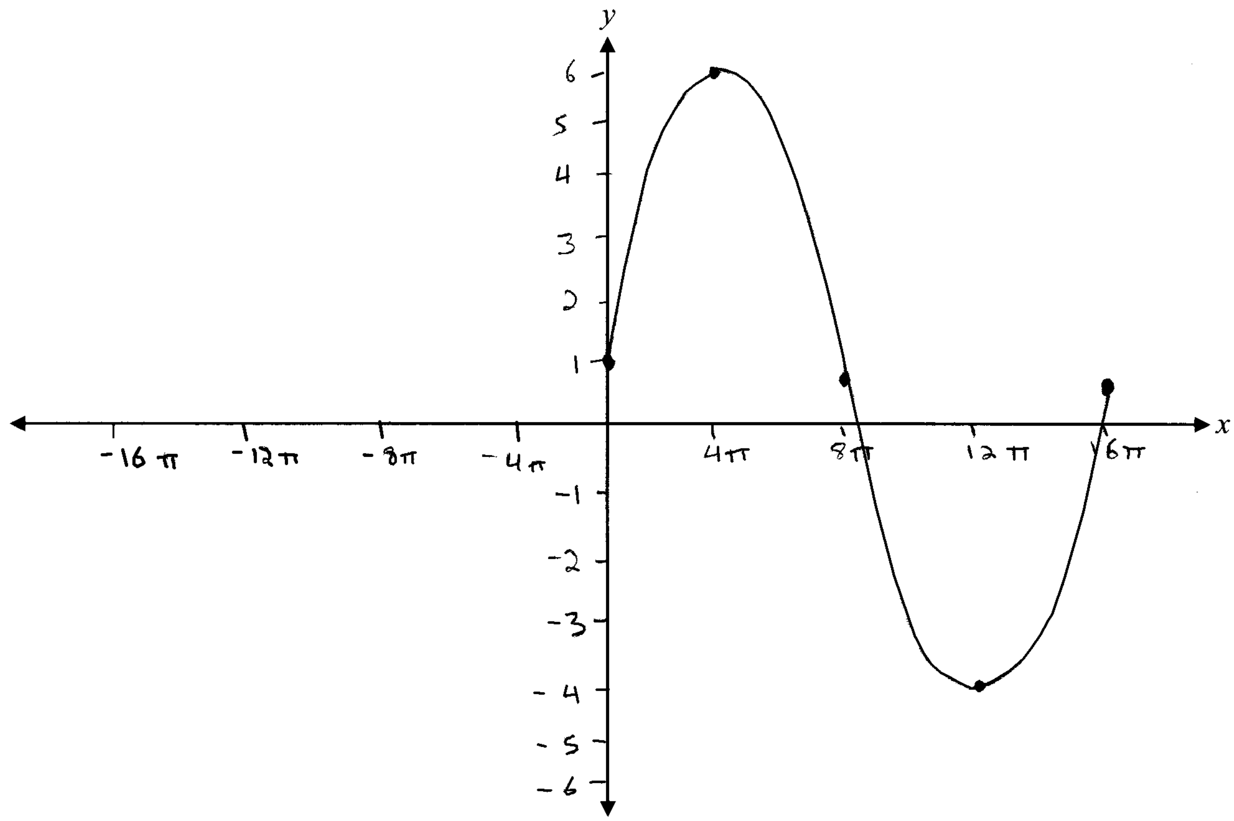
Exemplar 2



3 out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for vertical translation

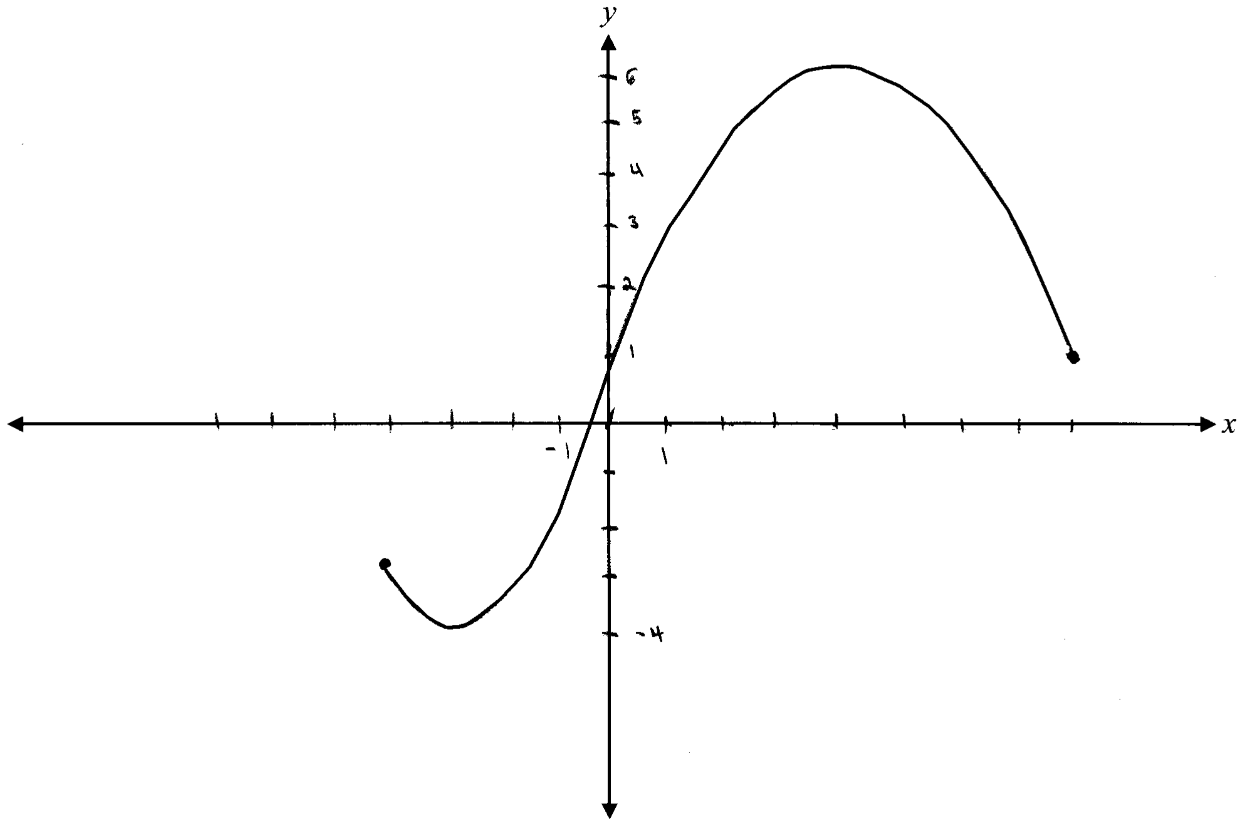
Exemplar 3



3 out of 4

+ 1 mark for shape of $y = \sin x$
+ 1 mark for amplitude
+ 1 mark for vertical translation

Exemplar 4



2 out of 4

+ 1 mark for amplitude
+ 1 mark for vertical translation

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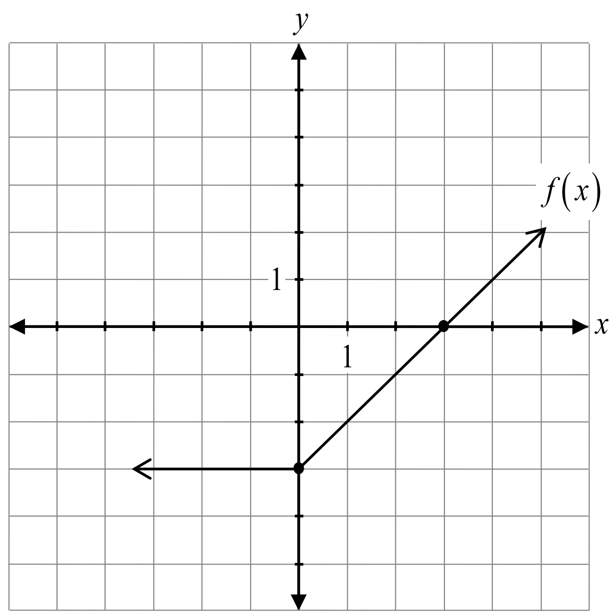
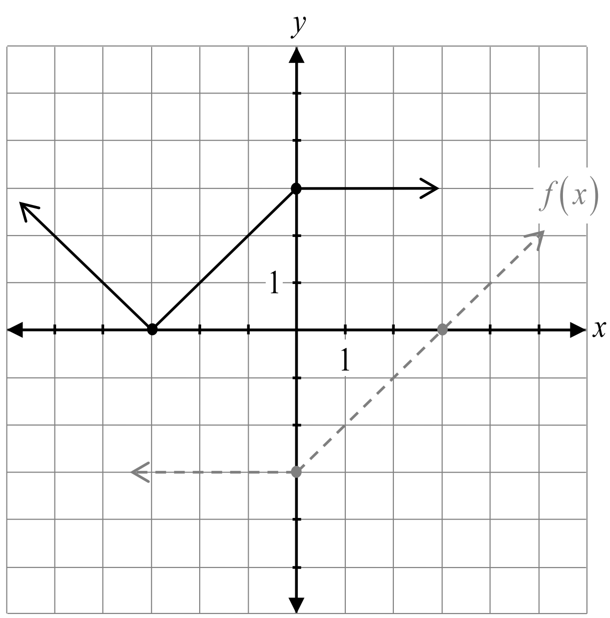
Question 43 R1, R5

Given the graph of $y = f(x)$, sketch the graph of $y = |f(-x)|$.



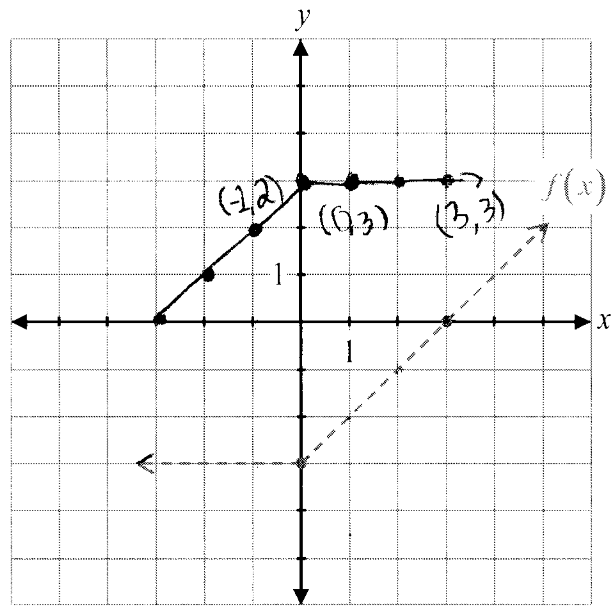
Solution



1 mark for horizontal reflection
1 mark for absolute value

2 marks

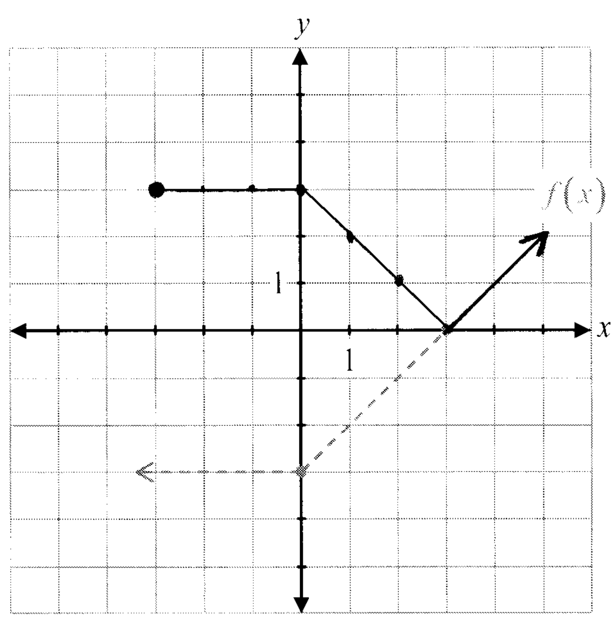
Exemplar 1



1 out of 2

+ 1 mark for horizontal reflection

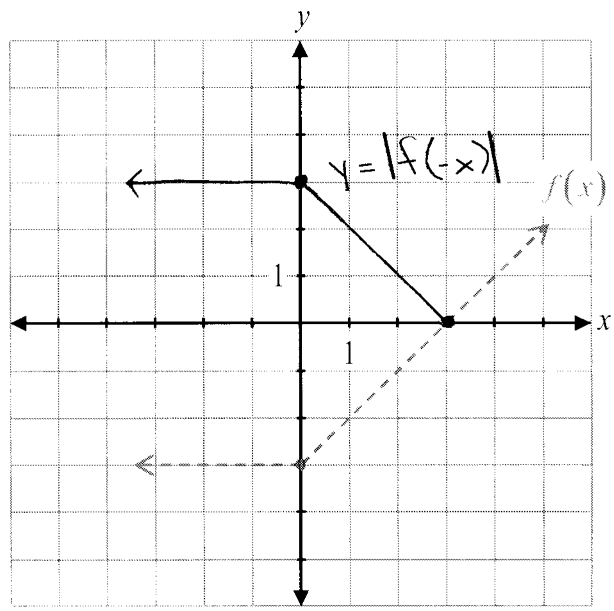
Exemplar 2



1 out of 2

+ 1 mark for absolute value
E9 (endpoints or arrowheads omitted or incorrect)

Exemplar 3



0 out of 2

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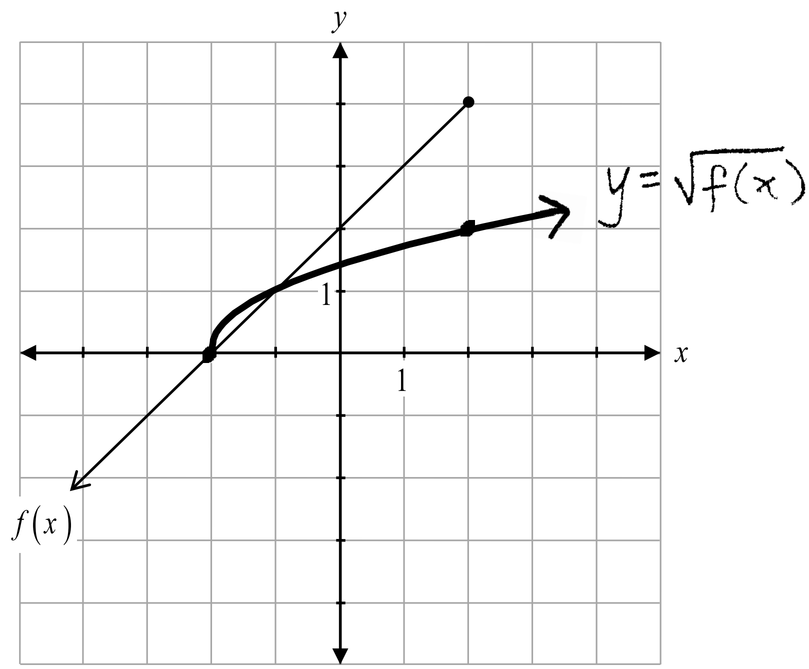
Savannah used the graph of $y = f(x)$ to sketch the graph of $y = \sqrt{f(x)}$. Her solution is given below. Describe her error.



Solution

Savannah did not restrict the domain at $x = 2$.

1 mark

Exemplar 1

her arrow is the wrong direction, you can't have it going on forever that way if f(x) ends at 2,4.

0 out of 1

Exemplar 2

She made the line continuous, but it ends at (2,4), so she should have just ended it there.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Question 45 T6

Determine the exact value of $\sin\left(\frac{13\pi}{12}\right)$.

Solution

$$\sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)=\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

1 mark for substitution into correct identity

$$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

2 marks (½ mark for each exact value)

$$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$

$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

3 marks

Note:

- Other combinations are possible.

Exemplar 1

$$\frac{13\cancel{\pi}}{\cancel{12}_{\cancel{2}_{1}}} \times \frac{\cancel{180}^{\cancel{30}^{15}}}{\cancel{\pi}} = (13)(15) = 195^\circ$$

$$\begin{array}{r} 15 \\ \times\ 13 \\ \hline 45 \\ +15\ \times \\ \hline 195 \end{array}$$

$$\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right) = \left(\sin\frac{3\pi}{4}\right)\left(\cos\frac{5\pi}{6}\right) + \left(\cos\frac{3\pi}{4}\right)\left(\sin\frac{5\pi}{6}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{-\sqrt{5}}{4} + \frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{5} + 2\sqrt{2}}{4}$$

$$\boxed{\sin\left(\frac{13\pi}{12}\right) = \frac{2\sqrt{2} - \sqrt{5}}{4}}$$

$$\frac{45}{180} = \frac{x}{\pi}$$

$$36x = 9\pi$$

$$x = \frac{3\pi}{4}$$

$$\begin{array}{r} 36 \\ 5\overline{)180} \\ -15\ 3 \\ 30 \end{array}$$

$$\frac{150}{180} = \frac{x}{\pi}$$

$$18x = 15\pi$$

$$x = \frac{5\pi}{6}$$

1½ out of 3

+ 1 mark for substitution into correct identity
+ ½ mark for exact value of $\sin\frac{3\pi}{4}$
+ ½ mark for exact value of $\cos\frac{5\pi}{6}$
+ ½ mark for exact value of $\sin\frac{5\pi}{6}$
– ½ mark for procedural error of incorrect combination
– ½ mark for arithmetic error in line 4

Exemplar 2

$$\sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) =$$

$$\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) =$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} =$$

$$\frac{2\sqrt{2}}{2}$$

0 out of 3

Exemplar 3

$$\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$$

$$= \sin\frac{\sqrt{2}}{2}\cos\frac{1}{2} + \cos\frac{-\sqrt{2}}{2}\sin\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

2½ out of 3

award full marks
– ½ mark for procedural error in line 2

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Question 46 R14

State the equation of the horizontal asymptote of $f(x) = \dfrac{3x}{x-1}$.

Solution

$y = 3$ **1 mark**

Exemplar 1

$y = 0$

0 out of 1

Exemplar 2

$x = 3$

0 out of 1

Exemplar 3

H.A. = 3

½ out of 1

award full marks
– ½ mark for procedural error

Sketch the graph of $f(x) = \dfrac{5x - 10}{x^2 + x - 6}$.

Solution

$$f(x) = \frac{5x - 10}{x^2 + x - 6}$$

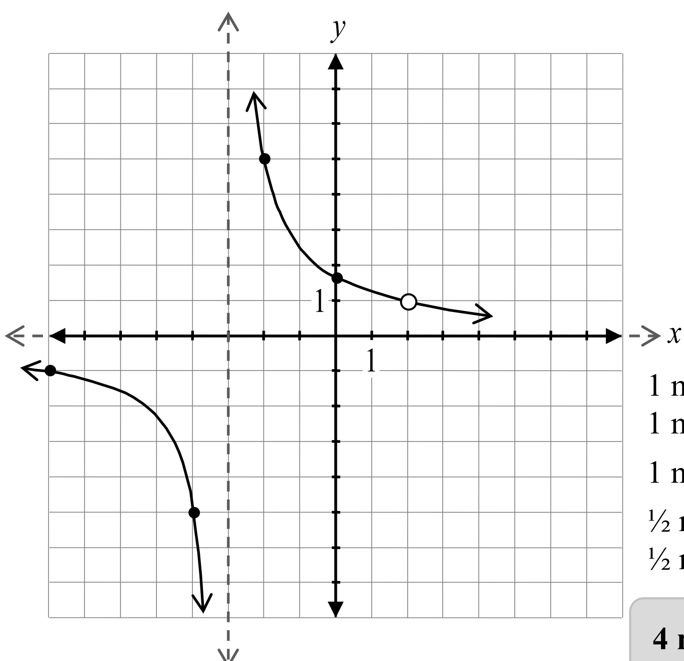
$$= \frac{5\cancel{(x-2)}}{\cancel{(x-2)}(x+3)}$$

$$= \frac{5}{x+3}, x \neq 2$$

$\therefore$ there is a point of discontinuity (hole) at $(2, 1)$

vertical asymptote at $x = -3$

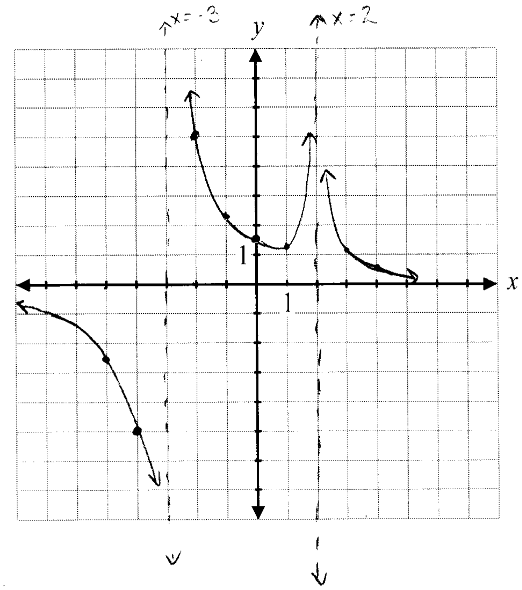
horizontal asymptote at $y = 0$



1 mark for asymptotic behaviour approaching $x = -3$
1 mark for asymptotic behaviour approaching $y = 0$
1 mark for a point of discontinuity (hole) at $(2, 1)$
½ mark for graph left of $x = -3$
½ mark for graph right of $x = -3$

4 marks

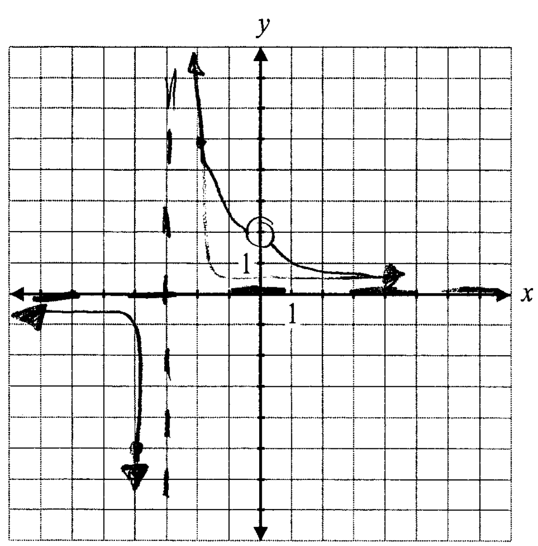
Exemplar 1



2½ out of 4

+ 1 mark for asymptotic behaviour approaching $x = -3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -3$
E10 (asymptote omitted but still implied)

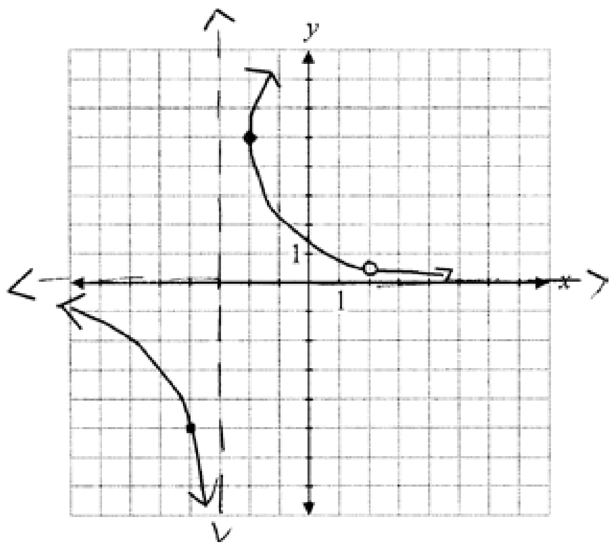
Exemplar 2



3 out of 4

+ 1 mark for asymptotic behaviour approaching $x = -3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -3$
+ ½ mark for graph right of $x = -3$

Exemplar 3



3½ out of 4

award full marks
– ½ mark for procedural error (incorrect *y*-value for point of discontinuity (hole))
E10 (graph curls away from asymptote)

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Question 48 R6

Determine, algebraically, the inverse of $f(x) = 3x + 4$.

Solution

Let $f(x) = y$	
$y = 3x + 4$	
$x = 3y + 4$	1 mark for switching x and y-values
$x - 4 = 3y$	
$\frac{x-4}{3} = y$	½ mark for solving for y
$f^{-1}(x) = \frac{x-4}{3}$	½ mark for writing equation of $f^{-1}(x)$

2 marks

Exemplar 1

$$x = 3y + 4$$

$$\frac{x-4}{3} = \frac{3y}{3}$$

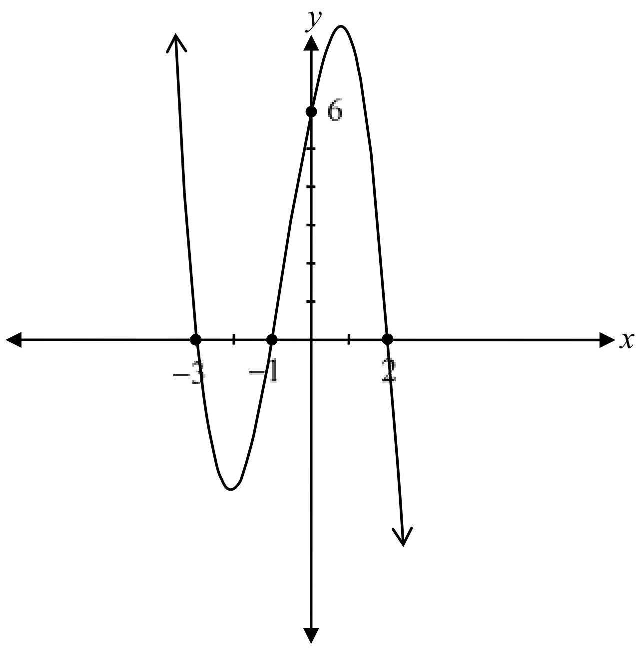
$$\boxed{f(x)^{-1} = \frac{x-4}{3}}$$

2 out of 2

award full marks
E7 (notation error in line 3)

Sketch the graph of $P(x) = -(x+1)(x-2)(x+3)$.

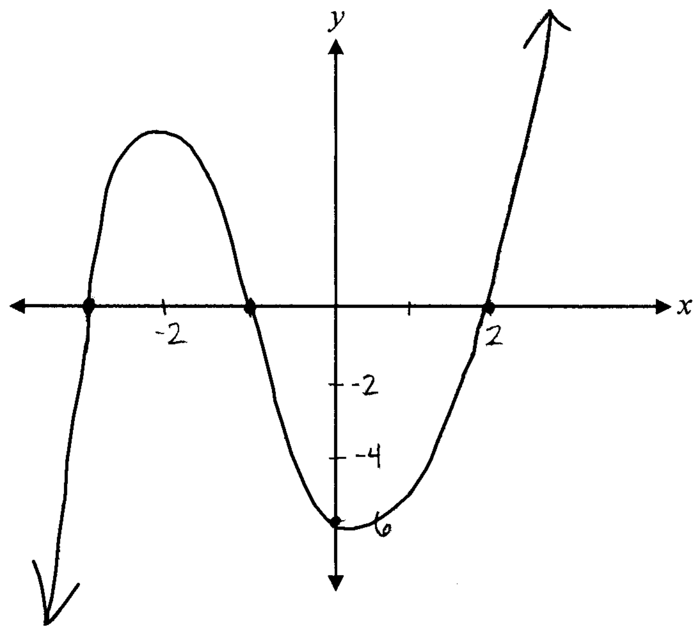
Solution



1 mark for x-intercepts
½ mark for y-intercept
½ mark for end behaviour

2 marks

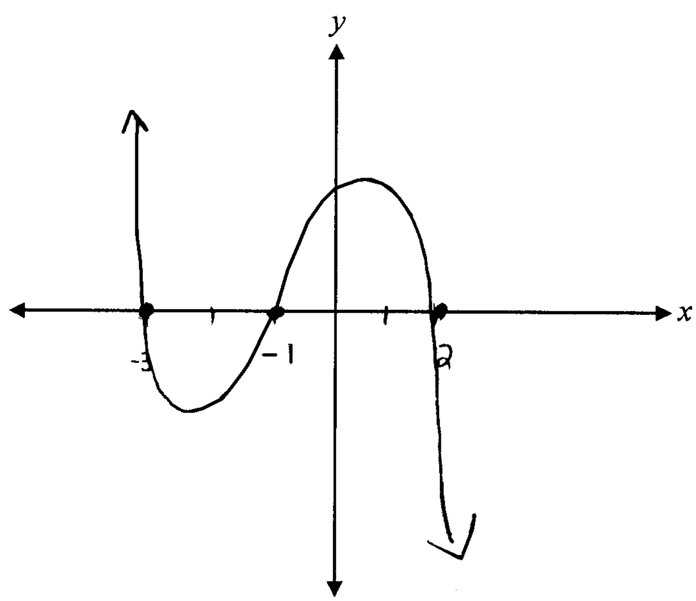
Exemplar 1



1 out of 2

+ 1 mark for x-intercepts

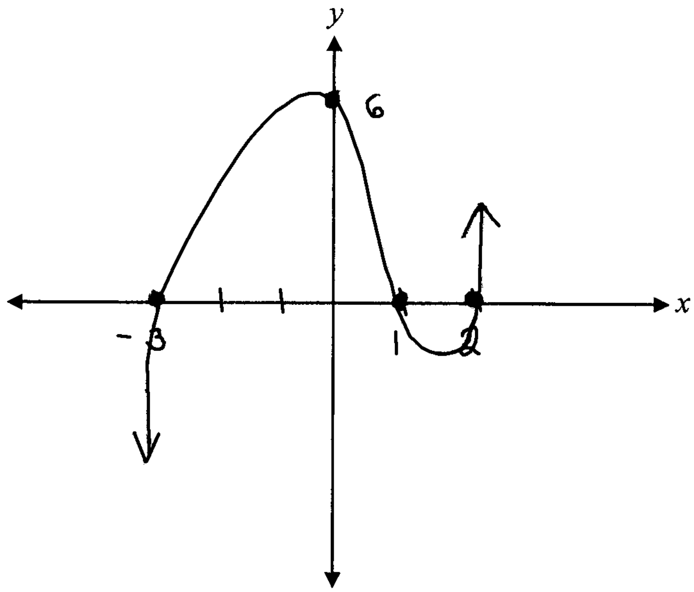
Exemplar 2



1½ out of 2

+ 1 mark for x-intercepts
+ ½ mark for end behaviour

Exemplar 3



1 out of 2

+ 1 mark for x-intercepts
+ ½ mark for y-intercept
– ½ mark for procedural error (one incorrect x-intercept)

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allowed for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

Question(s) affected: ______________________________

Action taken or rationale for assigning marks: ______________________________

Follow-up: ______________________________

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Decision: ______________________________

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Marker’s Signature: ______________________________

Principal’s Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
9	R4	2
11	R3	1
15	R5	1
17a)	R1	1
17b)	R1	1
21	R1	1
26	R1	1
27	R2	1
39	R1	1
43	R1, R5	2
48	R6	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
6	T1	1
13	T3	1
25	T1	1
29	T4	1
33	T2	2
40	T3	3
42	T4	4
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2	P1	1
4	P4	3
8	P4	1
16	P2	1
20	P3	1
23	P4	1
38	P3	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
7	R11	3
18	R11	1
30	R12	2
32	R12	1
49	R12	2

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
10	T6	3
22	T6	1
28	T6	2
31	T5	4
45	T6	3
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	3
5	R10	3
12	R8	3
19	R7	1
34	R9	3
36	R10	1
41	R10	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
14	R14	2
24	R13	1
35	R13	1
37	R13	3
44	R13	1
46	R14	1
47	R14	4