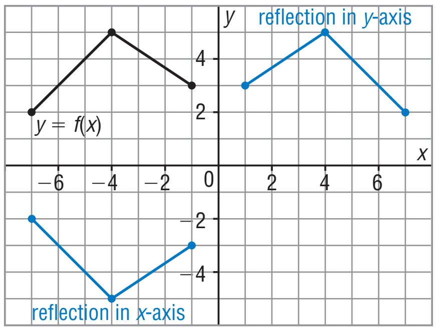
Lesson 3.2 Exercises, pages 183–190

A

3. Here is the graph of $y = f(x)$. On the same grid, sketch its image after each reflection.

a) a reflection in the x-axis

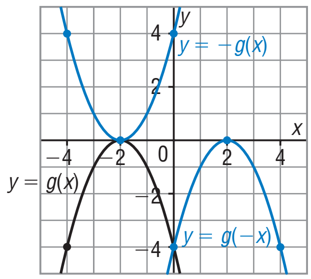
Reflect the endpoint of each line segment on $y = f(x)$ in the x-axis. Join corresponding points in order, to form the reflection image.



b) a reflection in the *y*-axis

Reflect the endpoint of each line segment on $y = f(x)$ in the y-axis. Join corresponding points in order, to form the reflection image.

4. Here is the graph of $y = g(x)$. On the same grid, sketch and label the graph of each function.



a) $y = -g(x)$

The graph of $y = -g(x)$ is the image of the graph of $y = g(x)$ after a reflection in the x-axis.
Choose lattice points on $y = g(x)$.

Point on $y = g(x)$	Point on $y = -g(x)$
(−4, −4)	(−4, 4)
(−2, 0)	(−2, 0)
(0, −4)	(0, 4)

Plot the points, then draw a smooth curve through them to form the graph of $y = -g(x)$.

b) $y = g(-x)$

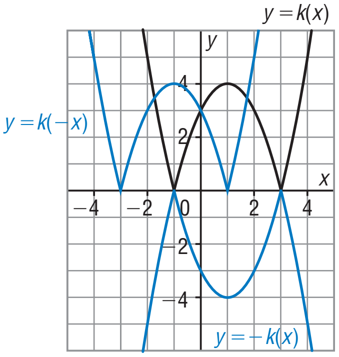
The graph of $y = g(-x)$ is the image of the graph of $y = g(x)$ after a reflection in the y-axis. Use the lattice points on $y = g(x)$.

Point on $y = g(x)$	Point on $y = g(-x)$
(−4, −4)	(4, −4)
(−2, 0)	(2, 0)
(0, −4)	(0, −4)

Plot the points, then draw a smooth curve through them to form the graph of $y = g(-x)$.

B

5. Here is the graph of $y = k(x)$. On the same grid, sketch and label the graph of each function below. State the domain and range of each function.



a) $y = -k(x)$

The graph of $y = -k(x)$ is the image of the graph of $y = k(x)$ after a reflection in the x-axis.
Choose lattice points on $y = k(x)$.

Point on $y = k(x)$	Point on $y = -k(x)$
$(-2, 5)$	$(-2, -5)$
$(-1, 0)$	$(-1, 0)$
$(1, 4)$	$(1, -4)$
$(3, 0)$	$(3, 0)$
$(4, 5)$	$(4, -5)$

Plot the points, then draw a smooth curve through them to form the graph of $y = -k(x)$.
Both functions have domain: $x \in \mathbb{R}$
Range of $y = k(x)$: $y \geq 0$; range of $y = -k(x)$: $y \leq 0$

b) $y = k(-x)$

The graph of $y = k(-x)$ is the image of the graph of $y = k(x)$ after a reflection in the y-axis. Use the lattice points on $y = k(x)$.

Point on $y = k(x)$	Point on $y = k(-x)$
$(-2, 5)$	$(2, 5)$
$(-1, 0)$	$(1, 0)$
$(1, 4)$	$(-1, 4)$
$(3, 0)$	$(-3, 0)$
$(4, 5)$	$(-4, 5)$

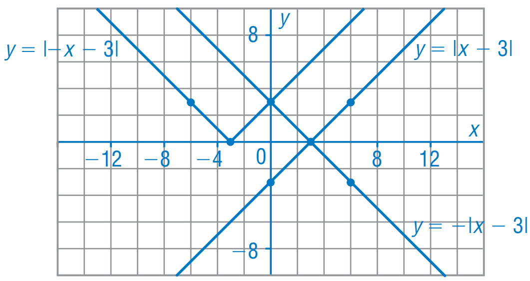
Plot the points, then draw a smooth curve through them to form the graph of $y = k(-x)$.
Both functions have domain: $x \in \mathbb{R}$
Both functions have range: $y \geq 0$

6. Sketch and label the graphs of each set of functions on the same grid. Describe the strategy you used. State the domain and range of each function.

a) $y = |x - 3|$ $\qquad y = -|x - 3|$ $\qquad y = |-x - 3|$

To graph $y = |x - 3|$, create a table of values including the x-intercept, plot the points, then join them with straight lines.

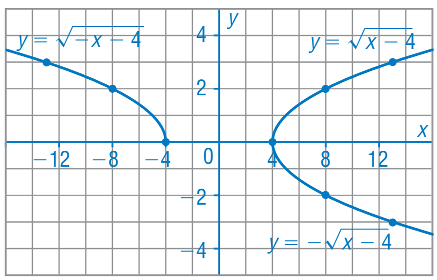
x	$y = \lvert x - 3\rvert$
0	3
3	0
6	3



To graph $y = -|x - 3|$, reflect the graph of $y = |x - 3|$ in the x-axis.
Since x was replaced with $-x$, to graph $y = |-x - 3|$, reflect the graph of $y = |x - 3|$ in the y-axis.
The domain of all functions is $x \in \mathbb{R}$.
The range of both $y = |x - 3|$ and $y = |-x - 3|$ is $y \geq 0$; the range of $y = -|x - 3|$ is $y \leq 0$.

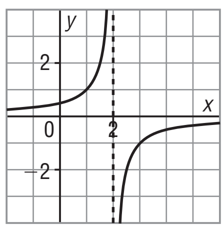
b) $y = \sqrt{x - 4}$ $\qquad y = \sqrt{-x - 4}$ $\qquad y = -\sqrt{x - 4}$

To graph $y = \sqrt{x - 4}$, translate the graph of $y = \sqrt{x}$ 4 units right.
Since x was replaced with $-x$, to graph $y = \sqrt{-x - 4}$, reflect the graph of $y = \sqrt{x - 4}$ in the y-axis. To graph $y = -\sqrt{x - 4}$, reflect the graph of $y = \sqrt{x - 4}$ in the x-axis.



The domain of both $y = \sqrt{x - 4}$ and $y = -\sqrt{x - 4}$ is $x \geq 4$;
the domain of $y = \sqrt{-x - 4}$ is $x \leq -4$.
The range of both $y = \sqrt{x - 4}$ and $y = \sqrt{-x - 4}$ is $y \geq 0$;
the range of $y = -\sqrt{x - 4}$ is $y \leq 0$.

7. The graph of $y = \dfrac{1}{x - 2}$ was reflected in the x-axis and its image is shown. What is an equation of the image?



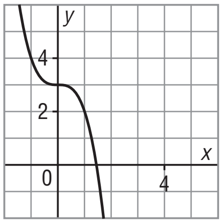
When the graph of $y = f(x)$ is reflected in the x-axis, the equation of its image is $y = -f(x)$.
So, an equation of the image is:

$$y = -f(x)$$

$$y = -\left(\frac{1}{x - 2}\right)$$

$$y = \frac{1}{-x + 2}$$

8. a) The graph of $y = x^3 + 3$ was reflected in the y-axis and its image is shown. What is an equation of the image? How can you verify your answer?



When the graph of $y = f(x)$ is reflected in the y-axis, the equation of its image is $y = f(-x)$.
So, an equation of the image is:
$y = f(-x)$
$y = (-x)^3 + 3$
$y = -x^3 + 3$
To verify, I graph both functions on a graphing calculator.

b) What is an equation of the image when the graph of $y = x^3 + 3$ is reflected in the x-axis?

When the graph of $y = f(x)$ is reflected in the x-axis, the equation of its image is $y = -f(x)$. So, an equation of the image is:
$y = -f(x)$
$y = -(x^3 + 3)$
$y = -x^3 - 3$

9. Write an equation for the image of each function below after:
i) a reflection of its graph in the x-axis
ii) a reflection of its graph in the y-axis

a) $g(x) = -2x^3 + x^2 - 5x - 3$

i) After a reflection in the x-axis, an equation of its image is:
$y = -g(x)$
$y = -(-2x^3 + x^2 - 5x - 3)$
$y = 2x^3 - x^2 + 5x + 3$
ii) After a reflection in the y-axis, an equation of its image is:
$y = g(-x)$
$y = -2(-x)^3 + (-x)^2 - 5(-x) - 3$
$y = 2x^3 + x^2 + 5x - 3$

b) $k(x) = \dfrac{1}{-(x + 3)^2 + 4}$

i) After a reflection in the x-axis, an equation of its image is:
$y = -k(x)$
$$y = -\left(\frac{1}{-(x + 3)^2 + 4}\right)$$
$$y = \frac{1}{(x + 3)^2 - 4}$$
ii) After a reflection in the y-axis, an equation of its image is:
$y = k(-x)$
$$y = \frac{1}{-(-x + 3)^2 + 4}$$

10. The graph of $y = f(x)$ has x-intercepts 5, 2, and –1, and y-intercept 10. What are the x-intercepts and y-intercept of the image graph after each reflection?

a) a reflection in the x-axis

After a reflection in the x-axis, the point (x, y) on $y = f(x)$ corresponds to the point $(x, -y)$ on the image graph.
The points (5, 0), (2, 0), (−1, 0), and (0, 10) on the graph of $y = f(x)$ correspond to the points (5, 0), (2, 0), (−1, 0), and (0, −10) on the image graph. So, the image graph has x-intercepts 5, 2, and −1, and y-intercept −10.

b) a reflection in the y-axis

After a reflection in the y-axis, the point (x, y) on $y = f(x)$ corresponds to the point $(-x, y)$ on the image graph.
The points (5, 0), (2, 0), (−1, 0), and (0, 10) on the graph of $y = f(x)$ correspond to the points (−5, 0), (−2, 0), (1, 0), and (0, 10) on the image graph. So, the image graph has x-intercepts −5, −2, and 1, and y-intercept 10.

11. The function $y = f(x)$ has domain $-2 \leq x \leq 8$ and range $6 \leq y \leq 20$. Determine the domain and range of each function.

a) $y = -f(x)$

The graph of $y = -f(x)$ is the image of the graph of $y = f(x)$ after a reflection in the x-axis. So, (x, y) on $y = f(x)$ corresponds to $(x, -y)$ on $y = -f(x)$. Since the x-values do not change, the domain of $y = -f(x)$ is $-2 \leq x \leq 8$. Since the y-values change sign, the range of $y = -f(x)$ is $-20 \leq y \leq -6$.

b) $y = f(-x)$

The graph of $y = f(-x)$ is the image of the graph of $y = f(x)$ after a reflection in the y-axis.
So, (x, y) on $y = f(x)$ corresponds to $(-x, y)$ on $y = f(-x)$.
Since the x-values change sign, the domain of $y = f(-x)$ is $-8 \leq x \leq 2$.
Since the y-values do not change, the range of $y = f(-x)$ is $6 \leq y \leq 20$.

12. Determine where the graph of $f(x) = x^2 + x - 6$ intersects the graph of each function below. How do you know?

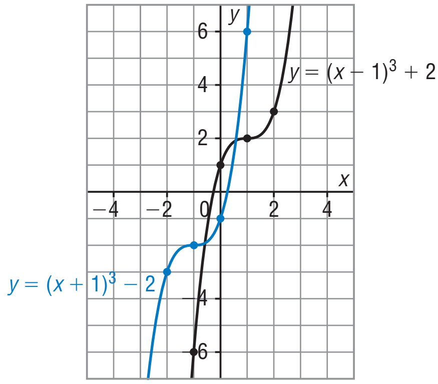
a) $y = -f(x)$

The graph of $y = -f(x)$ is the image of the graph of $y = f(x)$ after a reflection in the x-axis, so the x-intercepts, if they exist, are invariant.
Determine the x-intercepts of:
$y = x^2 + x - 6$ Substitute: $y = 0$
$0 = (x + 3)(x - 2)$
$x + 3 = 0$ or $x - 2 = 0$
$x = -3$ or $x = 2$
So, the graphs intersect at the points (−3, 0) and (2, 0).

b) $y = f(-x)$

When the graph of a function is reflected in the y-axis, the y-intercept, if it exists, is invariant.
From the equation, the y-intercept of the graph of $f(x) = x^2 + x - 6$ is -6.
So, the graphs intersect at the point $(0, -6)$.

13. Here is the graph of $y = (x - 1)^3 + 2$. On the same grid, sketch the graph of the final image after each pair of reflections.
Write the equation of the final image.



a) a reflection in the y-axis, followed by a reflection in the x-axis

After a reflection in the y-axis, the point (x, y) on $y = (x - 1)^3 + 2$ corresponds to the point $(-x, y)$ on $y = (-x - 1)^3 + 2$.
Then, after a reflection in the x-axis, the point $(-x, y)$ on $y = (-x - 1)^3 + 2$ corresponds to the point $(-x, -y)$ on $y = -(-x - 1)^3 - 2$, which simplifies to $y = (x + 1)^3 - 2$.
Choose lattice points on $y = (x - 1)^3 + 2$.

Point on $y = (x - 1)^3 + 2$	Point on $y = (x + 1)^3 - 2$
(x, y)	$(-x, -y)$
$(-1, -6)$	$(1, 6)$
$(0, 1)$	$(0, -1)$
$(1, 2)$	$(-1, -2)$
$(2, 3)$	$(-2, -3)$

Plot the points in the 2nd column, then draw a smooth curve through them to create the graph of $y = -(-x - 1)^3 - 2$.

b) a reflection in the x-axis, followed by a reflection in the y-axis

After a reflection in the x-axis, the point (x, y) on $y = (x - 1)^3 + 2$ corresponds to the point $(x, -y)$ on $y = -(x - 1)^3 - 2$.
Then, after a reflection in the y-axis, the point $(x, -y)$ on $y = -(x - 1)^3 - 2$ corresponds to the point $(-x, -y)$ on $y = -(-x - 1)^3 - 2$, which simplifies to $y = (x + 1)^3 - 2$. This is the same equation as in part a.

c) Does the order in which the reflections are performed matter? Explain.

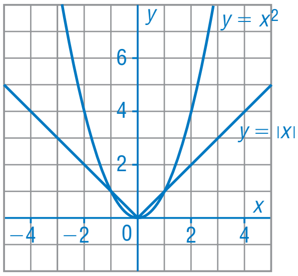
No, the order does not matter. The final images are the same and their equations are the same no matter in which order the reflections are performed.

C

14. A function $f(x)$ is an *even function* when $f(x) = f(-x)$.

a) Give 2 examples of even functions. Sketch their graphs.

Sample response: Two examples of even functions are $y = |x|$ and $y = x^2$.



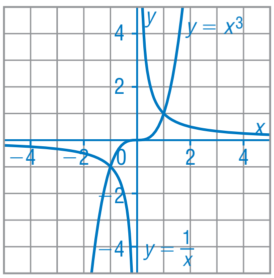
b) What property do all even functions share? Why?

The graph of an even function is symmetrical about the y-axis because the image of the point (x, y) is the point $(-x, y)$. This means that the graph does not change after a reflection in the y-axis.

15. A function $f(x)$ is an *odd function* when $f(-x) = -f(x)$.

a) Give 2 examples of odd functions. Sketch their graphs.

Sample response: Two examples of odd functions are $y = \frac{1}{x}$ and $y = x^3$.



b) What property do all odd functions share? Why?

The graph of an odd function has rotational symmetry of order 2 about the origin because the point (x, y) is reflected in each axis to get the final image point $(-x, -y)$. This means the graph does not change after a rotation of 180° about the origin.