Lesson 7.5 Exercises, pages 641–649

A

3. Verify each identity for the given values of α and β.

a) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$; for $\alpha = 90°$ and $\beta = 30°$

Substitute: $\alpha = 90°$ and $\beta = 30°$

$$\begin{aligned}\text{L.S.} &= \sin(\alpha + \beta)\\ &= \sin(90° + 30°)\\ &= \sin 120°\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ &= \sin 90°\cos 30° + \cos 90°\sin 30°\\ &= (1)\left(\frac{\sqrt{3}}{2}\right) + (0)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{3}}{2} + 0\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

The left side is equal to the right side, so the identity is verified.

b) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$; for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

Substitute: $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$$\begin{aligned}\text{L.S.} &= \cos(\alpha - \beta)\\ &= \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\\ &= \sin\frac{\pi}{4}\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= \cos\alpha\cos\beta + \sin\alpha\sin\beta\\ &= \cos\frac{\pi}{2}\cos\frac{\pi}{4} + \sin\frac{\pi}{2}\sin\frac{\pi}{4}\\ &= (0)\left(\frac{1}{\sqrt{2}}\right) + (1)\left(\frac{1}{\sqrt{2}}\right)\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

The left side is equal to the right side, so the identity is verified.

c) $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$; for $\alpha = \frac{2\pi}{3}$ and $\beta = \frac{\pi}{3}$

Substitute: $\alpha = \frac{2\pi}{3}$ and $\beta = \frac{\pi}{3}$

$$\begin{aligned}\text{L.S.} &= \tan(\alpha + \beta)\\ &= \tan\left(\frac{2\pi}{3} + \frac{\pi}{3}\right)\\ &= \tan\pi\\ &= 0\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\\ &= \frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{3}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{3}}\\ &= \frac{-\sqrt{3} + \sqrt{3}}{1 - (-\sqrt{3})(\sqrt{3})}\\ &= 0\end{aligned}$$

The left side is equal to the right side, so the identity is verified.

4. Simplify each expression.

a) $\cos 8\theta \cos 2\theta + \sin 8\theta \sin 2\theta$

$= \cos (8\theta - 2\theta)$
$= \cos 6\theta$

b) $\cos \theta \sin 4\theta - \sin \theta \cos 4\theta$

$= \sin 4\theta \cos \theta - \cos 4\theta \sin \theta$
$= \sin (4\theta - \theta)$
$= \sin 3\theta$

c) $\dfrac{\tan 7x - \tan 3x}{1 + \tan 7x \tan 3x}$

$= \tan (7x - 3x)$
$= \tan 4x$

d) $\sin 5x \sin 3x - \cos 5x \cos 3x$

$= -(\cos 5x \cos 3x - \sin 5x \sin 3x)$
$= -\cos (5x + 3x)$
$= -\cos 8x$

B

5. Simplify each expression, then determine its exact value.

a) $\cos 75° \cos 15° - \sin 75° \sin 15°$

$= \cos (75° + 15°)$
$= \cos 90°$
$= 0$

b) $\sin \pi \cos \frac{\pi}{4} + \cos \pi \sin \frac{\pi}{4}$

$= \sin \left(\pi + \frac{\pi}{4}\right)$
$= \sin \frac{5\pi}{4}$
$= -\frac{1}{\sqrt{2}}$

c) $\cos \frac{\pi}{3} \sin \frac{\pi}{6} - \sin \frac{\pi}{3} \cos \frac{\pi}{6}$

$= -\left(\sin \frac{\pi}{3} \cos \frac{\pi}{6} - \cos \frac{\pi}{3} \sin \frac{\pi}{6}\right)$
$= -\sin \left(\frac{\pi}{3} - \frac{\pi}{6}\right)$
$= -\sin \frac{\pi}{6}$
$= -\frac{1}{2}$

d) $\dfrac{\tan \pi + \tan \frac{\pi}{3}}{1 - \tan \pi \tan \frac{\pi}{3}}$

$= \tan \left(\pi + \frac{\pi}{3}\right)$
$= \tan \frac{4\pi}{3}$
$= \sqrt{3}$

6. a) Expand $\sin \left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ to verify that $\sin \pi = 0$.

$\sin \left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \sin \frac{\pi}{2} \cos \frac{\pi}{2} + \cos \frac{\pi}{2} \sin \frac{\pi}{2}$
$\sin \pi = (1)(0) + (0)(1)$
$= 0$

b) Expand $\cos \left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ to verify that $\cos \pi = -1$.

$\cos \left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} \cos \frac{\pi}{2} - \sin \frac{\pi}{2} \sin \frac{\pi}{2}$
$\cos \pi = (0)(0) - (1)(1)$
$= -1$

c) Expand $\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$ to verify that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

$$\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \cos\frac{\pi}{2}\cos\frac{\pi}{3} + \sin\frac{\pi}{2}\sin\frac{\pi}{3}$$
$$\cos\frac{\pi}{6} = (0)\left(\frac{1}{2}\right) + (1)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

d) Expand $\tan\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$ to verify that $\tan\frac{\pi}{3} = \sqrt{3}$.

$$\tan\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{6}}$$
$$\tan\frac{\pi}{3} = \frac{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}, \text{ or } \sqrt{3}$$

7. Determine each exact value.

a) $\cos 75°$

$$= \cos(45° + 30°)$$
$$= \cos 45° \cos 30° - \sin 45° \sin 30°$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

b) $\sin 15°$

$$= \sin(45° - 30°)$$
$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

c) $\tan\frac{\pi}{12}$

$$= \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
$$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$
$$= \frac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}$$
$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

d) $\tan\frac{5\pi}{12}$

$$= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$
$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$$
$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \left(\frac{1}{\sqrt{3}}\right)(1)}$$
$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

8. a) Given $\sin \beta = -\frac{1}{3}$ and $\cos \alpha = \frac{2}{5}$, where angle β is in standard position with its terminal arm in Quadrant 3 and angle α is in standard position with its terminal arm in Quadrant 4; determine each exact value.

i) $\sin(\alpha - \beta)$ **ii)** $\cos(\alpha - \beta)$

iii) $\tan(\alpha + \beta)$ **iv)** $\tan(\alpha - \beta)$

Use: $x^2 + y^2 = r^2$

For angle α,
substitute: $x = 2, r = 5$
$2^2 + y^2 = 5^2$
$y = -\sqrt{21}$ since the terminal arm of angle α lies in Quadrant 4.

So, $\sin \alpha = -\frac{\sqrt{21}}{5}$

For angle β,
substitute: $y = -1, r = 3$
$x^2 + (-1)^2 = 3^2$
$x = -\sqrt{8}$ since the terminal arm of angle β lies in Quadrant 3.

So, $\cos \beta = -\frac{\sqrt{8}}{3}$

Substitute for α and β in:

i) $\sin(\alpha - \beta)$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \left(-\frac{\sqrt{21}}{5}\right)\left(-\frac{\sqrt{8}}{3}\right) - \left(\frac{2}{5}\right)\left(-\frac{1}{3}\right)$$
$$= \frac{\sqrt{168}}{15} + \frac{2}{15}$$
$$= \frac{\sqrt{168} + 2}{15}$$

ii) $\cos(\alpha - \beta)$

$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \left(\frac{2}{5}\right)\left(-\frac{\sqrt{8}}{3}\right) + \left(-\frac{\sqrt{21}}{5}\right)\left(-\frac{1}{3}\right)$$
$$= \frac{-2\sqrt{8}}{15} + \frac{\sqrt{21}}{15}$$
$$= \frac{-2\sqrt{8} + \sqrt{21}}{15}$$

iii) $$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$= \frac{-\frac{\sqrt{21}}{2} + \frac{1}{\sqrt{8}}}{1 - \left(-\frac{\sqrt{21}}{2}\right)\left(\frac{1}{\sqrt{8}}\right)}$$
$$= \frac{-\sqrt{168} + 2}{2\sqrt{8} + \sqrt{21}}$$

iv) $$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$$
$$= \frac{\sqrt{168} + 2}{-2\sqrt{8} + \sqrt{21}}$$

b) What other strategy could you use to determine $\tan(\alpha - \beta)$?

Sample response: I could substitute for $\tan\alpha$ and $\tan\beta$ in the identity for $\tan(\alpha - \beta)$.

9. Prove each identity.

a) $\cos\theta = \sin\left(\frac{\pi}{2} + \theta\right)$

Use: $\sin(\alpha + \beta)$
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$
Substitute: $\alpha = \frac{\pi}{2}, \beta = \theta$

$$\begin{aligned}\text{R.S.} &= \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta\\ &= (1)\cos\theta + (0)\sin\theta\\ &= \cos\theta\\ &= \text{L.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

b) $-\tan\theta = \tan(\pi - \theta)$

Use: $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Substitute: $\alpha = \pi, \beta = \theta$

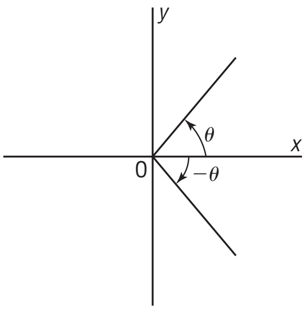
$$\begin{aligned}\text{R.S.} &= \frac{\tan\pi - \tan\theta}{1 + \tan\pi\tan\theta}\\ &= \frac{0 - \tan\theta}{1 + (0)\tan\theta}\\ &= -\tan\theta\\ &= \text{L.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

10. For each identity below:

i) Use the diagram at the right to explain why each identity is true.

ii) Prove the identity.



a) $\cos(-\theta) = \cos\theta$

i) The cosine of an angle whose terminal arm lies in Quadrant 4 is positive and equal to the cosine of its reference angle in Quadrant 1.

ii)
$$\begin{aligned}\text{L.S.} &= \cos(-\theta)\\ &= \cos(0 - \theta)\\ &= \cos 0\cos\theta + \sin 0\sin\theta\\ &= (1)\cos\theta + (0)\sin\theta\\ &= \cos\theta\\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

b) $\sin(-\theta) = -\sin\theta$

i) The sine of an angle whose terminal arm lies in Quadrant 4 is negative and has the same numerical value as the sine of its reference angle in Quadrant 1.

ii)
$$\begin{aligned}\text{L.S.} &= \sin(-\theta)\\ &= \sin(0 - \theta)\\ &= \sin 0\cos\theta - \cos 0\sin\theta\\ &= (0)\cos\theta - (1)\sin\theta\\ &= -\sin\theta\\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

c) $\tan(-\theta) = -\tan\theta$

i) The tangent of an angle whose terminal arm lies in Quadrant 4 is negative and has the same numerical value as the tangent of its reference angle in Quadrant 1.

ii) $$\begin{aligned}\text{L.S.} &= \tan(-\theta)\\ &= \frac{\sin(-\theta)}{\cos(-\theta)}\\ &= \frac{-\sin\theta}{\cos\theta}\\ &= -\tan\theta\\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

11. If $f(x)$ is an even function, then $f(-x) = f(x)$
If $f(x)$ is an odd function, then $f(-x) = -f(x)$
Use the identities in question 10 to classify each of the sine, cosine, and tangent functions as odd or even.

Since $\sin(-x) = -\sin x$, the sine function is an odd function
Since $\cos(-x) = \cos x$, the cosine function is an even function
Since $\tan(-x) = -\tan x$, the tangent function is an odd function

12. Solve this equation over the domain $-90° < x < 270°$:
$$\frac{\tan 4x - \tan 3x}{1 + \tan 4x \tan 3x} = \sqrt{3}$$

$$\begin{aligned}\tan(4x - 3x) &= \sqrt{3}\\ \tan x &= \sqrt{3}\\ x = 60° \text{ or } x &= 240°\end{aligned}$$

13. Given $\tan\alpha = \frac{4}{3}$ and $\tan\beta = -\frac{5}{12}$, where angle α is in standard position in Quadrant 1 and angle β is in standard position with its terminal arm in Quadrant 2, determine the exact value of $\cos(\alpha - \beta)$.

Use: $x^2 + y^2 = r^2$

For angle α, substitute:
$x = 3, y = 4$
$3^2 + 4^2 = r^2$
$r = 5$
So, $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$

For angle β, substitute:
$x = -12, y = 5$
$(-12)^2 + 5^2 = r^2$
$r = 13$
So, $\sin\beta = \frac{5}{13}$ and $\cos\beta = -\frac{12}{13}$

Substitute for α and β in:
$$\begin{aligned}\cos(\alpha - \beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta\\ &= \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)\\ &= \frac{-36}{65} + \frac{20}{65}\\ &= -\frac{16}{65}\end{aligned}$$

14. Prove each identity.

a) $\sin\left(\frac{\pi}{4} + \theta\right) + \sin\left(\frac{\pi}{4} - \theta\right) = \sqrt{2}\cos\theta$

$$\begin{aligned}\text{L.S.} &= \sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta \\ &= 2\sin\frac{\pi}{4}\cos\theta \\ &= 2\left(\frac{1}{\sqrt{2}}\right)\cos\theta \\ &= \sqrt{2}\cos\theta \\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

b) $\cos\left(\frac{\pi}{6} + \theta\right) - \cos\left(\frac{\pi}{6} - \theta\right) = -\sin\theta$

$$\begin{aligned}\text{L.S.} &= \cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta - \left(\cos\frac{\pi}{6}\cos\theta + \sin\frac{\pi}{6}\sin\theta\right) \\ &= -2\sin\frac{\pi}{6}\sin\theta \\ &= -2\left(\frac{1}{2}\right)\sin\theta \\ &= -\sin\theta \\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

c) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$$\begin{aligned}\text{L.S.} &= \tan(\alpha - \beta) \\ &= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} && \text{Expand.} \\ &= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} && \text{Divide numerator and denominator by } \cos\alpha\cos\beta. \\ &= \frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} && \text{Simplify.} \\ &= \frac{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{\sin\beta}{\cos\beta}}{1 + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} && \text{Use the tangent identity.} \\ &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \\ &= \text{R.S.}\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

15. Solve each equation over the domain $-\frac{3\pi}{2} \leq x < \frac{\pi}{2}$.

a) $\sin 7x \cos 5x - \cos 7x \sin 5x = -1$

$$\sin(7x - 5x) = -1$$
$$\sin 2x = -1$$

Consider $-3\pi \leq 2x < \pi$.

$2x = -\frac{\pi}{2}$ or $2x = -\frac{5\pi}{2}$

$x = -\frac{\pi}{4}$ $\quad$ $x = -\frac{5\pi}{4}$

The roots are: $x = -\frac{\pi}{4}$ and $x = -\frac{5\pi}{4}$

b) $\cos 2x \cos x - \sin 2x \sin x = 0$

$$\cos(2x + x) = 0$$
$$\cos 3x = 0$$

Consider $-\frac{9\pi}{2} \leq 3x < \frac{3\pi}{2}$.

$3x = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ or $-\frac{3\pi}{2}$ or $-\frac{5\pi}{2}$ or $-\frac{7\pi}{2}$ or $-\frac{9\pi}{2}$

$x = \frac{\pi}{6}$ or $-\frac{\pi}{6}$ or $-\frac{\pi}{2}$ or $-\frac{5\pi}{6}$ or $-\frac{7\pi}{6}$ or $-\frac{3\pi}{2}$

The roots are: $x = \pm\frac{\pi}{6}, x = -\frac{\pi}{2}, x = -\frac{5\pi}{6}, x = -\frac{7\pi}{6}, x = -\frac{3\pi}{2}$

16. Here are two solutions for solving the equation $\sin(\pi + x) = \frac{1}{\sqrt{2}}$ over the set of real numbers. Are both solutions correct? Explain.

Solution 1

$\sin(\pi + x) = \frac{1}{\sqrt{2}}$

Either $\pi + x = \frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}$

$x = -\frac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$

Or $\pi + x = \frac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$

$x = -\frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}$

Solution 2

$\sin(\pi + x) = \frac{1}{\sqrt{2}}$

$\sin \pi \cos x + \cos \pi \sin x = \frac{1}{\sqrt{2}}$

$(0)\cos x + (-1)\sin x = \frac{1}{\sqrt{2}}$

$-\sin x = \frac{1}{\sqrt{2}}$

$\sin x = -\frac{1}{\sqrt{2}}$

$x = \frac{5\pi}{4} + 2\pi k, k \in \mathbb{Z}$

or $x = \frac{7\pi}{4} + 2\pi k, k \in \mathbb{Z}$

Both solutions are correct. For $k = 1$, $-\frac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}$ becomes $x = \frac{5\pi}{4}$, and $-\frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}$ becomes $\frac{7\pi}{4}$.

17. Determine the general solution of each equation over the set of real numbers.

a) $\cos(\pi + x) = \frac{1}{2}$

$\cos \pi \cos x - \sin \pi \sin x = \frac{1}{2}$

$(-1)\cos x - (0)\sin x = \frac{1}{2}$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}$ or

$x = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$

b) $\tan(\pi + x) = -1$

$\frac{\tan \pi + \tan x}{1 - \tan \pi \tan x} = -1$

$\frac{0 + \tan x}{1 - (0)\tan x} = -1$

$\tan x = -1$

$x = \frac{3\pi}{4} + \pi k, k \in \mathbb{Z}$

C

18. Use algebra to determine the amplitude and the period of the graph of $y = \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$. Describe your strategy. Use graphing technology to check.

$y = \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$

Use the sum and difference formulas.

$y = \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$

$y = 2\sin\frac{\pi}{4}\cos x$

$y = 2\left(\frac{1}{\sqrt{2}}\right)\cos x$

$y = \sqrt{2}\cos x$

The amplitude is $\sqrt{2}$ and the period is 2π.