PRACTICE TEST, pages 453–456

1. Multiple Choice Which logarithm has the greatest value?

A. $\log_3 30$ **(B.)** $\log_2 9$ **C.** $\log_4 60$ **D.** $\log_7 360$

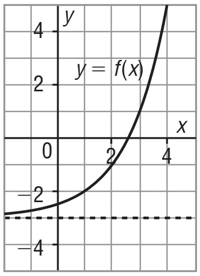
2. Multiple Choice Which function describes this graph?

A. $f(x) = 2^{x+1} + 3$

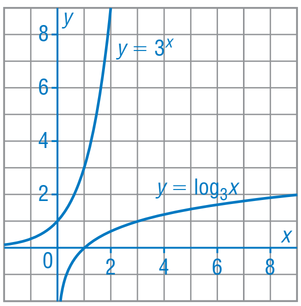
(B.) $f(x) = 2^{x-1} - 3$

C. $f(x) = 2^{x+1} - 3$

D. $f(x) = 2^{x-1} + 3$



3. a) Graph $y = 3^x$. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.



Make a table of values, plot the points, then join them with a smooth curve.

x	y
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9

There is no x-intercept; the y-intercept is 1; the equation of the asymptote is $y = 0$; the domain is $x \in \mathbb{R}$; and the range is $y > 0$.

b) Sketch the graph of $y = \log_3 x$ on the grid in part a. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

Interchange the coordinates in part a, plot the points, then join them with a smooth curve.
There is no y-intercept; the x-intercept is 1; the equation of the asymptote is $x = 0$; the domain is $x > 0$; and the range is $y \in \mathbb{R}$.

c) How do the graphs in parts a and b show that these functions are inverses?

Each graph is a reflection of the other in the line $y = x$.

4. Solve each equation. Where necessary, give the solution to the nearest hundredth.

a) $5^x = 400$

$$\log_5 5^x = \log_5 400$$
$$x = \frac{\log 400}{\log 5}$$
$$x \doteq 3.72$$

b) $8^{x+1} = 16^{x-2}$

$$2^{3(x+1)} = 2^{4(x-2)}$$
$$3x + 3 = 4x - 8$$
$$x = 11$$

c) $\log_4(x - 8) + \log_4(x + 4) = 3$

$x > 8$ and $x > -4$; so $x > 8$

$$\log_4(x - 8)(x + 4) = 3$$
$$(x - 8)(x + 4) = 4^3$$
$$x^2 - 4x - 96 = 0$$
$$(x - 12)(x + 8) = 0$$

$x = 12$ or $x = -8$

$x = -8$ is extraneous.

Verify $x = 12$:

$$\text{L.S.} = \log_4 4 + \log_4 16$$
$$= 1 + 2$$
$$= 3$$
$$= \text{R.S.}$$

The solution is verified.

d) $\log_2(3x + 4) = \log_2(x + 4) + \log_2(x - 2)$

$x > -\frac{4}{3}$, $x > -4$, and $x > 2$; so $x > 2$

$$\log_2(3x + 4) = \log_2(x + 4)(x - 2)$$
$$3x + 4 = (x + 4)(x - 2)$$
$$x^2 - x - 12 = 0$$
$$(x - 4)(x + 3) = 0$$

$x = 4$ or $x = -3$

$x = -3$ is extraneous.

Verify $x = 4$:

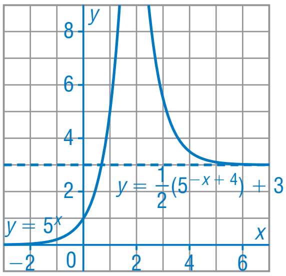
$$\text{L.S.} = \log_2 16$$
$$\text{R.S.} = \log_2 8 + \log_2 2$$
$$= \log_2 16$$

The solution is verified.

5. a) Graph $y = 5^x$.

Make a table of values, plot the points, then join them with a smooth curve.

x	y
-2	$\frac{1}{25}$
-1	$\frac{1}{5}$
0	1
1	5



b) Which transformations would be applied to the graph of $y = 5^x$ so that the equation of its image is $y = \frac{1}{2}(5^{-x+4}) + 3$?

Write the function as: $y - 3 = \frac{1}{2}(5^{-(x-4)})$

The transformations are: a vertical compression by a factor of $\frac{1}{2}$; a reflection in the y-axis; a horizontal translation of 4 units right; and a vertical translation of 3 units up

c) Graph $y = \frac{1}{2}(5^{-x+4}) + 3$ on the grid in part a. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

Compare: $y - 3 = \frac{1}{2}(5^{-(x-4)})$ with $y - k = c5^{d(x-h)}$:

$k = 3, c = \frac{1}{2}, d = -1, h = 4$

Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = 5^x$ corresponds to the point $\left(-x + 4, \frac{1}{2}y + 3\right)$ on $y - 3 = \frac{1}{2}(5^{-(x-4)})$. Use the points from part a.

x	6	5	4	3
y	3.02	3.1	3.5	5.5

There is no x-intercept.
For the y-intercept, substitute $x = 0$ in $y = \frac{1}{2}(5^{-x+4}) + 3$

$$y = \frac{1}{2}(5^4) + 3$$
$$y = 315.5$$

The equation of the asymptote is $y = 3$.
The domain is $x \in \mathbb{R}$ and the range is $y > 3$.

6. How would you add two logarithms with different bases? Include an example in your explanation.

I would write each logarithm to base 10, then use a calculator.

For example, $\log_2 9 + \log_3 11 = \frac{\log 9}{\log 2} + \frac{\log 11}{\log 3}$

$= 5.3525...$

7. How many monthly investments of $150 would have to be paid into a savings account that pays 3% annual interest, compounded monthly, to obtain an amount of $5000?

The amount of $5000 is the future value, so use:

$$FV = \frac{R[(1 + i)^n - 1]}{i} \quad \text{Substitute: } FV = 5000;\ R = 150;\ i = \frac{0.03}{12}, \text{ or } 0.0025$$

$$5000 = \frac{150[(1 + 0.0025)^n - 1]}{0.0025}$$

$$\frac{0.25}{3} = 1.0025^n - 1$$

$$1 + \frac{0.25}{3} = 1.0025^n$$

$$\log\left(\frac{3.25}{3}\right) = \log 1.0025^n$$

$$\log\left(\frac{3.25}{3}\right) = n \log 1.0025$$

$$n = \frac{\log\left(\frac{3.25}{3}\right)}{\log 1.0025}$$

$$n = 32.0570...$$

The number of investments is approximately 32.