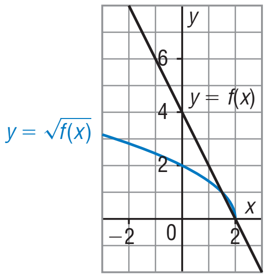
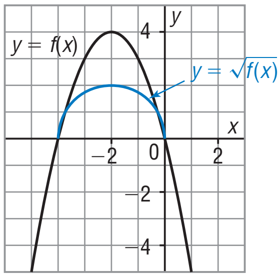
REVIEW, pages 146–153

2.1

1. a) For each graph of $y = f(x)$ below:

- Sketch the graph of $y = \sqrt{f(x)}$.
- State the domain and range of $y = \sqrt{f(x)}$.

i) 

ii) 

i) Mark points where $y = 0$ or $y = 1$. The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$ between these points. Choose, then mark other points.

x	$y = f(x)$	$y = \sqrt{f(x)}$
–2	8	$\doteq$ 2.8
0	4	2

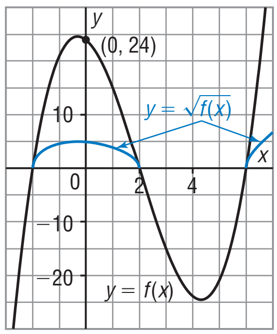
Join all points with a smooth curve.
The domain is: $x \leq 2$
The range is: $y \geq 0$

ii) $y = \sqrt{f(x)}$ is not defined for $x < -4$ or $x > 0$.
Mark points where $y = 0$ or $y = 1$. Choose, then mark another point.

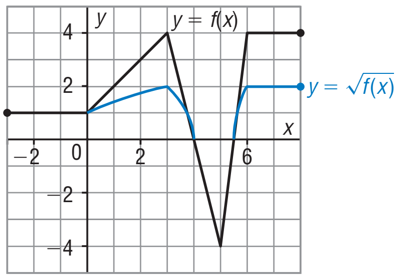
x	$y = f(x)$	$y = \sqrt{f(x)}$
–2	4	2

Join the points with a smooth curve.
The domain is: $-4 \leq x \leq 0$
The range is: $y \geq 0$

iii)



iv)



iii) The domain of $y = \sqrt{f(x)}$ is: $-2 \leq x \leq 2, x \geq 6$
Mark points where $y = 0$ or $y = 1$. Identify the coordinates of other points.

x	$y = f(x)$	$y = \sqrt{f(x)}$
0	24	$\doteq 5$
6.5	20	$\doteq 4.5$

Join the points with 2 smooth curves, for the graph of $y = \sqrt{f(x)}$.
The range of $y = \sqrt{f(x)}$ is: $y \geq 0$

iv) Mark points where $y = 0$ or $y = 1$. Identify and mark the coordinates of other points above the x-axis.

x	$y = f(x)$	$y = \sqrt{f(x)}$
3	4	2
6	4	2

Join the points with 2 smooth curves; except where the graph of $y = f(x)$ is horizontal, then the graph of $y = \sqrt{f(x)}$ is also horizontal.
The domain is: $-3 \leq x \leq 4$, $5.5 \leq x \leq 8$
The range is: $0 \leq y \leq 2$

b) Choose one pair of graphs from part a for which the domains are different and the ranges are different.

- Explain the strategy you used to graph the radical function.
- Explain why the domains differ and the ranges differ.

Sample response for part a) ii: I marked points where $y = 0$ or $y = 1$ because these points are invariant and lie on the graphs of both $y = f(x)$ and $y = \sqrt{f(x)}$. The graph of $y = \sqrt{f(x)}$ lies above the graph of $y = f(x)$ between these points because the square root of a number between 0 and 1 is greater than the number. I determined the coordinates of another point to improve my sketch.
The domains differ because $y = f(x)$ is defined for all real values of x while $y = \sqrt{f(x)}$ is only defined for values of x for which $f(x) \geq 0$.
The ranges differ because $f(x)$ can have any value less than or equal to 4, while $\sqrt{f(x)}$ can only be 0 or positive.

2. Use graphing technology to graph the functions $y = x^2 - 9$ and $y = \sqrt{x^2 - 9}$ on the same screen.

a) State the domain and range of the function $y = \sqrt{x^2 - 9}$.

The domain is: $x \leq -3, x \geq 3$
The range is: $y \geq 0$

b) How is the domain of the function $y = \sqrt{x^2 - 9}$ related to the domain of $y = x^2 - 9$?

The domain of $y = x^2 - 9$ is all real numbers. The domain of $y = \sqrt{x^2 - 9}$ is the real values of x for which $x^2 - 9 \geq 0$.

c) How are the zeros of $y = \sqrt{x^2 - 9}$ related to the zeros of $y = x^2 - 9$? Explain why.

The zeros of both functions are the same because their graphs have the same x-intercepts, and because the points with y-coordinate 0 are invariant.

d) What are the coordinates of the points of intersection of the two graphs? Explain your answer.

The graphs intersect at the invariant points; that is, where $y = 0$ and $y = 1$.
The coordinates of these points are: $(\pm 3, 0)$; $(\pm\sqrt{10}, 1)$, or approximately $(\pm 3.2, 1)$.

3. Solve each radical equation by graphing. Give the solution to the nearest tenth.

a) $x + 3 = \sqrt{5 - 2x}$

Write the equation as:
$x + 3 - \sqrt{5 - 2x} = 0$
Graph the related function:
$f(x) = x + 3 - \sqrt{5 - 2x}$
The approximate zero is:
-0.5358984
So, the solution is: $x \doteq -0.5$

b) $\sqrt{4x} = \sqrt{3x + 1} - x$

Write the equation as:
$\sqrt{4x} - \sqrt{3x + 1} + x = 0$
Graph the related function:
$f(x) = \sqrt{4x} - \sqrt{3x + 1} + x$
The approximate zero is:
0.29025371
So, the solution is: $x \doteq 0.3$

2.2

4. Without graphing each rational function below, predict whether its graph has a hole and/or any horizontal or vertical asymptotes. State the related non-permissible values.

a) $y = \frac{x^2}{x^2 - 5x + 4}$

Factor the denominator:

$y = \frac{x^2}{(x - 1)(x - 4)}$

The non-permissible values are $x = 1$ and $x = 4$; these are the equations of the vertical asymptotes. Since the degrees of the numerator and denominator are equal, there is a horizontal asymptote.

b) $y = \frac{x^2 - 5x + 4}{x - 4}$

Factor the numerator:

$y = \frac{(x - 1)(x - 4)}{x - 4}$

The non-permissible value is $x = 4$.

The numerator and denominator have a common factor, so there is a hole at $x = 4$. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

c) $y = \frac{x^2 - 4x - 5}{x - 1}$

Factor the numerator:

$y = \frac{(x - 5)(x + 1)}{x - 1}$

The non-permissible value is $x = 1$; this is the equation of the vertical asymptote. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

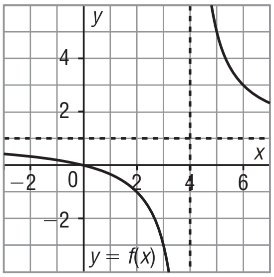
d) $y = \frac{x - 4}{x - 1}$

The non-permissible value is $x = 1$; this is the equation of the vertical asymptote. Since the degrees of the numerator and denominator are equal, there is a horizontal asymptote.

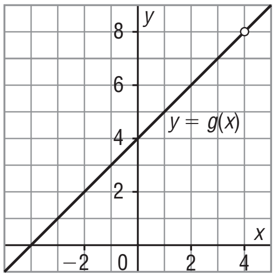
2.3

5. Match each function to its graph. Justify your choice.

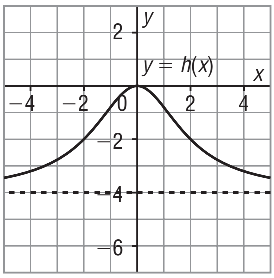
i) Graph A



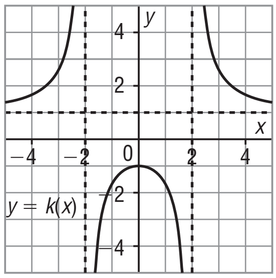
ii) Graph B



iii) Graph C



iv) Graph D



a) $y = \frac{-4x^2}{x^2 + 4}$

The denominator is always positive, so there is no vertical asymptote. The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the x-axis. The function matches Graph C.

b) $y = \frac{x}{x - 4}$

The function is undefined when $x = 4$, so this is the equation of the vertical asymptote. The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the x-axis. The function matches Graph A.

c) $y = \frac{x^2 + 4}{x^2 - 4}$

The function is undefined when $x^2 - 4 = 0$, so $x = \pm 2$; these are the equations of the vertical asymptotes. The degrees of the numerator and denominator are equal, so there is a horizontal asymptote that is not the x-axis. The function matches Graph D.

d) $y = \frac{x^2 - 16}{x - 4}$

Factor: $y = \frac{(x - 4)(x + 4)}{x - 4}$

There is a common factor $(x - 4)$, so there is a hole at $x = 4$. The function matches Graph B.

6. Solve each rational equation by graphing. Give the solution to the nearest tenth where necessary.

a) $\frac{2x - 1}{x + 3} = \frac{4}{x - 2}$

Graph a related function:
$f(x) = \frac{2x - 1}{x + 3} - \frac{4}{x - 2}$
Use graphing technology to determine the zeros:
$x \doteq -0.9$ or $x \doteq 5.4$

b) $\frac{x^2}{x^2 - 3} - \frac{x}{x + 2} = \frac{x}{2 - x}$

Graph a related function:
$f(x) = \frac{x^2}{x^2 - 3} - \frac{x}{x + 2} - \frac{x}{2 - x}$
Use graphing technology to determine the zeros:
$x \doteq -4.3$ or $x \doteq -1.5$ or $x = 0$, or $x \doteq 1.8$

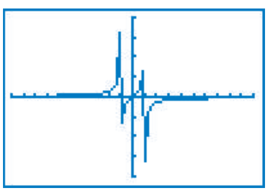
7. For the graph of each function below

i) Without graphing:
- Determine the coordinates of any holes and the equations of any asymptotes.
- Determine the domain.

ii) Use graphing technology to verify the characteristics and to explain the behaviour of the graph near the non-permissible values.

a) $y = \frac{-2x}{x^2 - 1}$

i) The function is undefined when $x^2 - 1 = 0$, so $x = \pm 1$; these are the equations of the vertical asymptotes.
Since the degree of the numerator is less than the degree of the denominator, there is a horizontal asymptote with equation $y = 0$.
The domain is: $x \neq \pm 1$

ii) 

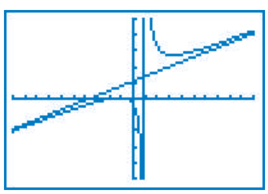
From the calculator screen: as $|x| \to \infty$, $y \to 0$, which verifies the horizontal asymptote; as $x \to \pm 1$, $y \to \pm\infty$, which verifies the vertical asymptotes.

b) $y = \frac{3x^2 + 8x + 4}{x - 1}$

i) The function is undefined when $x = 1$.
There are no common factors so $x = 1$ is the equation of the vertical asymptote.
There is an oblique asymptote.
Determine:
$(3x^2 + 8x + 4) \div (x - 1)$

1	3	8	4
		3	11
	3	11	15

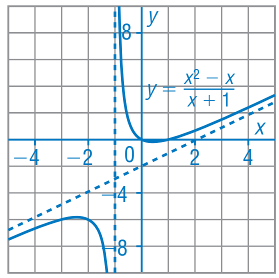
The quotient is $3x + 11$, so the equation of the oblique asymptote is $y = 3x + 11$.
The domain is: $x \neq 1$

ii) 

From the calculator screen: as $|x| \to \infty$, $y \to 3x + 11$, which verifies the oblique asymptote; as $x \to 1$, $y \to \pm\infty$, which verifies the vertical asymptote.

8. Without using graphing technology, sketch a graph of each function. State the domain.

a) $y = \frac{x^2 - x}{x + 1}$



The function is undefined when $x = -1$.

Factor: $y = \frac{x(x - 1)}{x + 1}$

There are no common factors, so there is a vertical asymptote with equation: $x = -1$

There is also an oblique asymptote. Determine: $(x^2 - x) \div (x + 1)$

$$\begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & & -1 & 2 \\ \hline & 1 & -2 & 2 \end{array}$$

The quotient is $x - 2$; so the equation of the oblique asymptote is $y = x - 2$.

Choose points, including those close to the vertical asymptote:

x	−2	−3	−1.01	−0.99
y	−6	−6	−203	197

Some of the y-values above are approximate.

When $x = 0$, $y = 0$

When $y = 0$,

$x^2 - x = 0$

$x(x - 1) = 0$

$x = 0$ or $x = 1$

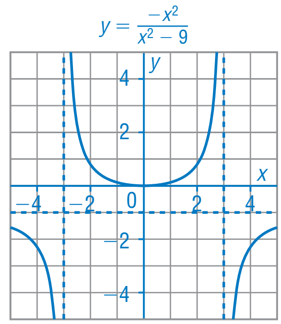
Plot points at these intercepts.

Draw broken lines for the asymptotes.

Join the points to form 2 smooth curves.

The domain is: $x \neq -1$

b) $y = \dfrac{-x^2}{x^2 - 9}$



The function is undefined when:

$x^2 - 9 = 0$

$x = \pm 3$

There are no common factors so there are vertical asymptotes with equations $x = \pm 3$.

There is a horizontal asymptote. The leading coefficients are −1 and 1, so the horizontal asymptote has equation $y = -1$.

Determine the behaviour of the graph close to the asymptotes.

x	−3.01	−2.99	2.99	3.01	−100	100
y	−151	149	149	−151	−1.001	−1.001

Some of the y-values above are approximate.

When $x = 0$, $y = 0$

When $y = 0$, $x = 0$

Plot a point at this intercept.

Determine the approximate coordinates of other points:

(−4, −2.3) and (4, −2.3)

Draw broken lines for the asymptotes, then sketch 3 smooth curves.

The domain is: $x \neq \pm 3$