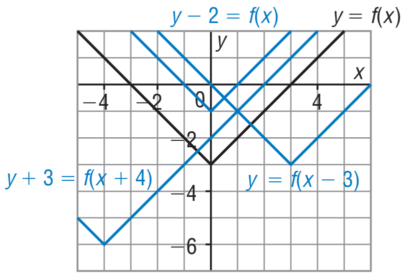
REVIEW, pages 255–260

3.1

1. Here is the graph of $y = f(x)$. Sketch the graph of each function below. Write the domain and range of each translation image.



a) $y - 2 = f(x)$

Each point on the graph of $y = f(x)$ is translated 2 units up. From the graph, the domain is: $x \in \mathbb{R}$; the range is: $y \geq -1$

b) $y = f(x - 3)$

Each point on the graph of $y = f(x)$ is translated 3 units right. From the graph, the domain is: $x \in \mathbb{R}$; the range is: $y \geq -3$

c) $y + 3 = f(x + 4)$

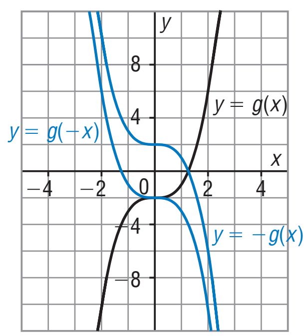
Each point on the graph of $y = f(x)$ is translated 4 units left and 3 units down. From the graph, the domain is: $x \in \mathbb{R}$; the range is: $y \geq -6$

2. The graph of the function $y = x^3 - 2$ is translated 3 units left and 4 units up. Write the equation of the translation image.

The equation of the translation image has the form $y - k = (x - h)^3 - 2$, with $h = -3$ and $k = 4$
So, the equation is: $y - 4 = (x + 3)^3 - 2$, which simplifies to $y = (x + 3)^3 + 2$

3.2

3. Here is the graph of $y = g(x)$. On the same grid, sketch the graph of each function below. Write the domain and range of each reflection image.



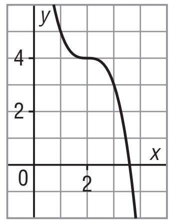
a) $y = -g(x)$

Reflect points on $y = g(x)$ in the x-axis:
$(-2, -10)$ becomes $(-2, 10)$
$(0, -2)$ becomes $(0, 2)$
$(2, 6)$ becomes $(2, -6)$
Draw a smooth curve through the points for the graph of $y = -g(x)$.
The domain is: $x \in \mathbb{R}$
The range is: $y \in \mathbb{R}$

b) $y = g(-x)$

Reflect points on $y = g(x)$ in the y-axis:
$(-2, -10)$ becomes $(2, -10)$
$(0, -2)$ stays as $(0, -2)$
$(2, 6)$ becomes $(-2, 6)$
Draw a smooth curve through the points for the graph of $y = g(-x)$.
The domain is: $x \in \mathbb{R}$
The range is: $y \in \mathbb{R}$

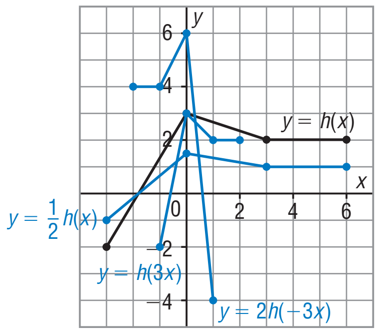
4. The graph of $f(x) = (x - 2)^3 - 4$ was reflected in the x-axis and its image is shown. What is an equation of the image?



When the graph of $y = f(x)$ is reflected in the x-axis, the equation of the image is $y = -f(x)$. So, an equation of the image is:
$f(x) = -[(x - 2)^3 - 4]$
$f(x) = -(x - 2)^3 + 4$

3.3

5. Here is the graph of $y = h(x)$. Sketch the graph of each function below. Write the domain and range of each transformation image.



a) $y = h(3x)$

The graph of $y = h(x)$ is compressed horizontally by a factor of $\frac{1}{3}$.
For each point at the ends of the line segments on $y = h(x)$, divide the x-coordinate by 3, plot the new points then join them for the graph of $y = h(3x)$.
The domain is: $-1 \le x \le 2$
The range is: $-2 \le y \le 3$

b) $y = \frac{1}{2}h(x)$

The graph of $y = h(x)$ is compressed vertically by a factor of $\frac{1}{2}$.
For each point at the ends of the line segments on $y = h(x)$, divide the y-coordinate by 2, plot the new points then join them for the graph of $y = \frac{1}{2}h(x)$.
The domain is: $-3 \le x \le 6$
The range is: $-1 \le y \le 1.5$

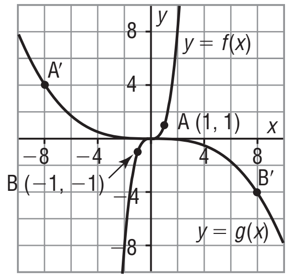
c) $y = 2h(-3x)$

The graph of $y = h(x)$ is stretched vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, then reflected in the y-axis.
Use: (x, y) on $y = h(x)$ corresponds to $\left(-\frac{x}{3}, 2y\right)$ on $y = 2h(-3x)$

Point on $y = h(x)$	Point on $y = 2h(-3x)$
$(-3, -2)$	$(1, -4)$
$(0, 3)$	$(0, 6)$
$(3, 2)$	$(-1, 4)$
$(6, 2)$	$(-2, 4)$

Plot the points, then join them.
The domain is: $-2 \le x \le 1$
The range is: $-4 \le y \le 6$

6. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



The graph has not been translated, so an equation of the image graph has the form: $y = af(bx)$

A point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b}, ay\right)$ on $y = af(bx)$.

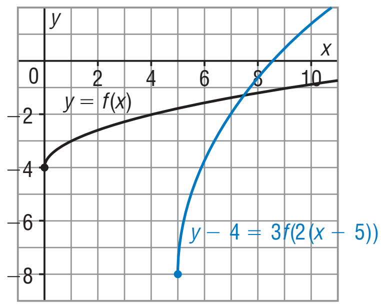
The image of A(1, 1) is $\left(\frac{1}{b}, 1a\right)$, which is A′(−8, 4).

Compare coordinates: $b = -\frac{1}{8}$ and $a = 4$

An equation for $y = g(x)$ is: $y = 4f\left(-\frac{1}{8}x\right)$

3.4

7. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of $y - 4 = 3f(2(x - 5))$. Write the domain and range of the transformation image.

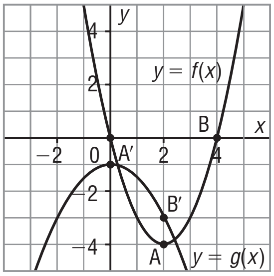


Compare: $y - k = af(b(x - h))$ to $y - 4 = 3f(2(x - 5))$
$k = 4$, $a = 3$, $b = 2$, and $h = 5$
A point (x, y) on the graph of $y = f(x)$ corresponds to the point $\left(\frac{x}{2} + 5, 3y + 4\right)$ on the graph of $y - 4 = 3f(2(x - 5))$.

Point on $y = f(x)$	Point on $y - 4 = 3f(2(x - 5))$
(0, −4)	(5, −8)
(1, −3)	(5.5, −5)
(4, −2)	(7, −2)
(9, −1)	(9.5, 1)

Plot the points, then join them.
From the graph of $y - 4 = 3f(2(x - 5))$, the domain is: $x \geq 5$; and the range is: $y \geq -8$

8. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



The equation of the image graph can be written as: $y - k = af(b(x - h))$
The horizontal distance between A and B is 2.
The vertical distance between A and B is 4.
The horizontal distance between A′ and B′ is 2.
The vertical distance between A′ and B′ is 2.
The graph of $y = f(x)$ has been compressed vertically by a factor of $\frac{1}{2}$ and reflected in the x-axis, so $a = -\frac{1}{2}$.
There is no horizontal stretch or compression, so $b = 1$.
Since B(4, 0) lies on the x-axis, it will not move after the vertical compression and reflection.
Determine the translation that would move B(4, 0) to B′(2, −3).
A translation of 2 units left and 3 units down is required, so $h = -2$ and $k = -3$
An equation for the image graph is: $y + 3 = -\frac{1}{2}f(x + 2)$

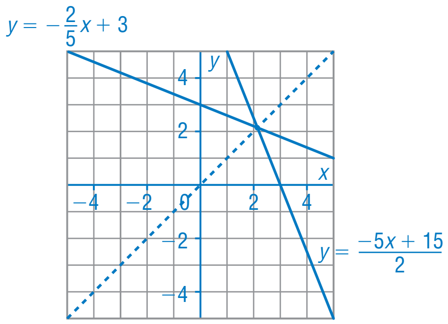
9. The point (2, 2) lies on the graph of $y = \frac{1}{4}x^3$. After a combination of transformations, the equation of the image graph is $y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right)$. What are the coordinates of the point that is the image of (2, 2)?

Compare: $y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right)$ with $y - k = af(b(x - h))$:
$k = -6$, $a = 5$, $b = 2$, and $h = 3$
A point (x, y) on the graph of $y = \frac{1}{4}x^3$ corresponds to the point $\left(\frac{x}{2} + 3, 5y - 6\right)$ on the graph of $y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right)$.
Substitute $x = 2$ and $y = 2$ in the expression for the coordinates above.
$\left(\frac{2}{2} + 3, 5(2) - 6\right) = (4, 4)$
The image of (2, 2) has coordinates (4, 4).

3.5

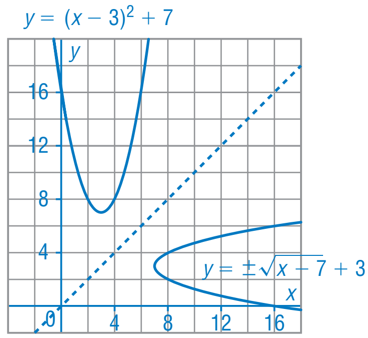
10. Determine an equation of the inverse of each function, then sketch graphs of the function and its inverse.

a) $y = -\frac{2}{5}x + 3$



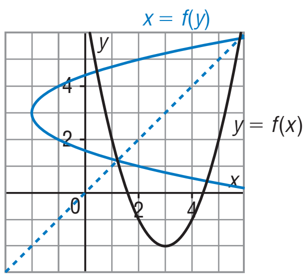
Write: $x = -\frac{2}{5}y + 3$
Solve for y.
$5x = -2y + 15$
$2y = -5x + 15$
$y = \frac{-5x + 15}{2}$
The graph of $y = -\frac{2}{5}x + 3$ is a line with y-intercept 3 and slope $-\frac{2}{5}$.
Reflect points on the graph of $y = -\frac{2}{5}x + 3$ in the line $y = x$.
Join the points for the graph of $y = \frac{-5x + 15}{2}$.

b) $y = (x - 3)^2 + 7$



Write: $x = (y - 3)^2 + 7$
Solve for y.
$(y - 3)^2 = x - 7$
$y - 3 = \pm\sqrt{x - 7}$
$y = \pm\sqrt{x - 7} + 3$
The graph of $y = (x - 3)^2 + 7$ is the image of the graph of $y = x^2$ after a translation of 3 units right and 7 units up.
Reflect points on the graph of $y = (x - 3)^2 + 7$ in the line $y = x$.
Join the points for the graph of $y = \pm\sqrt{x - 7} + 3$.

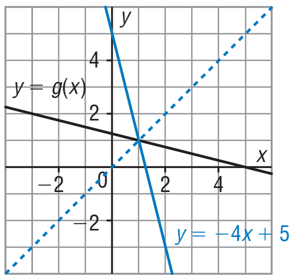
11. Restrict the domain of the function $y = f(x)$ so its inverse is a function.



Sample response: Sketch the graph of the inverse by reflecting points in the line $y = x$.
The inverse is a function if the domain of $y = f(x)$ is restricted to $x \leq 3$ or $x \geq 3$.

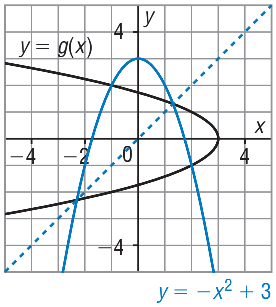
12. A graph was reflected in the line $y = x$. Its reflection image $y = g(x)$ is shown. Determine an equation of the original graph in terms of x and y.

a)



Use the line $y = x$ to sketch the graph of the inverse. This line has y-intercept 5, and slope -4, so its equation is: $y = -4x + 5$

b)



Use the line $y = x$ to sketch the graph of the inverse. This curve is a parabola that has vertex (0, 3), and is congruent to $y = -x^2$. So, its equation is: $y = -x^2 + 3$