REVIEW, pages 560–566

6.1

1. Determine the value of each trigonometric ratio. Use exact values where possible; otherwise write the value to the nearest thousandth.

a) tan (−45°)

$= -1$

b) cos 600°

$= \cos 240°$

$= -\frac{1}{2}$

c) sec (−210°)

$= \frac{1}{\cos(-210°)}$

$= \frac{1}{\cos 150°}$

$= -\frac{2}{\sqrt{3}}$

d) sin 765°

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$

e) cot 21°

$= \frac{1}{\tan 21°}$

$\doteq 2.605$

f) csc 318°

$= \frac{1}{\sin 318°}$

$\doteq -1.494$

2. To the nearest degree, determine all possible values of θ for which $\cos\theta = 0.76$, when $-360° \leq \theta \leq 360°$.

Since $\cos\theta$ is positive, the terminal arm of angle θ lies in Quadrant 1 or 4.
The reference angle is: $\cos^{-1}(0.76) \doteq 41°$
For the domain $0° \leq \theta \leq 360°$:
In Quadrant 1, $\theta \doteq 41°$
In Quadrant 4, $\theta \doteq 360° - 41°$, or approximately 319°
For the domain $-360° \leq \theta \leq 0°$:
In Quadrant 1, $\theta \doteq -360° + 41°$, or approximately −319°
In Quadrant 4, $\theta \doteq -41°$

6.2

3. As a fraction of π, determine the length of the arc that subtends a central angle of 225° in a circle with radius 3 units.

Arc length: $\frac{225}{360}(2\pi)(3) = \frac{15}{4}\pi$

6.3

4. a) Convert each angle to degrees. Give the answer to the nearest degree where necessary.

i) $\frac{5\pi}{3}$

$= \frac{5(180°)}{3}$

$= 300°$

ii) $-\frac{10\pi}{7}$

$= -\frac{10(180°)}{7}$

$\doteq -257°$

iii) 4

$= 4\left(\frac{180°}{\pi}\right)$

$\doteq 229°$

b) Convert each angle to radians.

i) 150°

$= 150\left(\frac{\pi}{180}\right)$

$= \frac{5\pi}{6}$

ii) −240°

$= -240\left(\frac{\pi}{180}\right)$

$= -\frac{4\pi}{3}$

iii) 485°

$= 485\left(\frac{\pi}{180}\right)$

$= \frac{97\pi}{36}$

5. In a circle with radius 5 cm, an arc of length 6 cm subtends a central angle. What is the measure of this angle in radians, and to the nearest degree?

Angle measure is: $\frac{\text{arc length}}{\text{radius}} = \frac{6}{5}$

$= 1.2$

In degrees, $1.2 = 1.2\left(\frac{180°}{\pi}\right)$

$\doteq 69°$

The angle measure is 1.2 radians or approximately 69°.

6. A race car is travelling around a circular track at an average speed of 120 km/h. The track has a diameter of 1 km. Visualize a line segment joining the race car to the centre of the track. Through what angle, in radians, will the segment have rotated in 10 s?

In 1 s, the car travels: $\frac{120}{60 \cdot 60}$ km $= \frac{1}{30}$ km

So, in 10 s, the car travels: $\frac{10}{30}$ km $= \frac{1}{3}$ km

Angle measure is: $\frac{\text{arc length}}{\text{radius}} = \frac{\frac{1}{3}}{1}$

$= \frac{1}{3}$

In 10 s, the segment will have rotated through an angle of $\frac{1}{3}$ radian.

7. Determine the value of each trigonometric ratio. Use exact values where possible; otherwise write the value to the nearest thousandth.

a) $\sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

b) $\cos \frac{5\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

c) $\sec\left(-\frac{\pi}{2}\right)$

$= \frac{1}{\cos\left(-\frac{\pi}{2}\right)}$, which is undefined

d) $\tan \frac{15\pi}{4}$

$= \tan \frac{3\pi}{4}$

$= -1$

e) csc 5

$= \frac{1}{\sin 5}$

$\doteq -1.043$

f) cot (−22.8)

$= \frac{1}{\tan(-22.8)}$

$\doteq -0.954$

8. P(3, −1) is a terminal point of angle θ in standard position.

a) Determine the exact values of all the trigonometric ratios for θ.

Let the distance between the origin and P be r.
Use: $x^2 + y^2 = r^2$ Substitute: $x = 3, y = -1$
$9 + 1 = r^2$
$r = \sqrt{10}$

$\sin\theta = -\frac{1}{\sqrt{10}}$ $\csc\theta = -\sqrt{10}$ $\cos\theta = \frac{3}{\sqrt{10}}$

$\sec\theta = \frac{\sqrt{10}}{3}$ $\tan\theta = -\frac{1}{3}$ $\cot\theta = -3$

b) To the nearest tenth of a radian, determine possible values of θ in the domain $-2\pi \leq \theta \leq 2\pi$.

The terminal arm of angle θ lies in Quadrant 4.
The reference angle is: $\tan^{-1}\left(\frac{1}{3}\right) = 0.3217\ldots$
So, $\theta = -0.3217\ldots$
The angle between 0 and 2π that is coterminal with $-0.3217\ldots$ is:
$2\pi - 0.3217\ldots = 5.9614\ldots$
Possible values of θ are approximately: 6.0 and −0.3

6.4

9. Use graphing technology to graph each function below for $-2\pi \leq x \leq 2\pi$, then list these characteristics of the graph: amplitude, period, zeros, domain, range, and the equations of the asymptotes.

a) $y = \sin x$

The amplitude is 1. The period is 2π. The zeros are $0, \pm\pi, \pm 2\pi$. The domain is $-2\pi \leq x \leq 2\pi$. The range is $-1 \leq y \leq 1$. There are no asymptotes.

b) $y = \cos x$

The amplitude is 1. The period is 2π. The zeros are $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$.
The domain is $-2\pi \leq x \leq 2\pi$. The range is $-1 \leq y \leq 1$. There are no asymptotes.

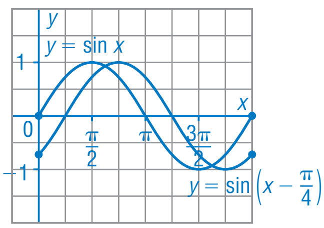
c) $y = \tan x$

There is no amplitude. The period is π. The zeros are $0, \pm\pi, \pm 2\pi$.
The domain is $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$. The range is $y \in \mathbb{R}$. The equations of the asymptotes are $x = \pm\frac{\pi}{2}$ and $x = \pm\frac{3\pi}{2}$.

6.5

10. On the same grid, sketch graphs of the functions in each pair for $0 \leq x \leq 2\pi$, then describe your strategy.

a) $y = \sin x$ and $y = \sin\left(x - \frac{\pi}{4}\right)$

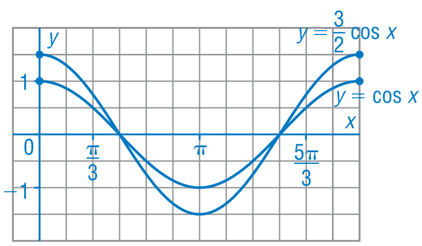


For the graph of $y = \sin x$, I used the completed table of values from Lesson 6.4.

The horizontal scale is 1 square to $\frac{\pi}{4}$ units, because the phase shift is $\frac{\pi}{4}$.

I then shifted several points $\frac{\pi}{4}$ units right and joined the points to get the graph of $y = \sin\left(x - \frac{\pi}{4}\right)$.

b) $y = \cos x$ and $y = \frac{3}{2}\cos x$

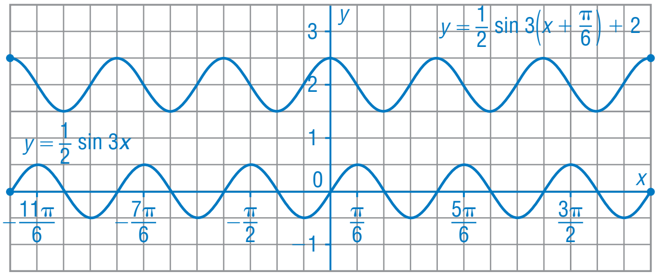


For the graph of $y = \cos x$, I used the completed table of values from Lesson 6.4.
I multiplied every y-coordinate by 1.5, plotted the new points, then joined them to get the graph of $y = \frac{3}{2}\cos x$.

6.6

11. a) Graph $y = \frac{1}{2} \sin 3\left(x + \frac{\pi}{6}\right) + 2$ for $-2\pi \leq x \leq 2\pi$.

Explain your strategy.

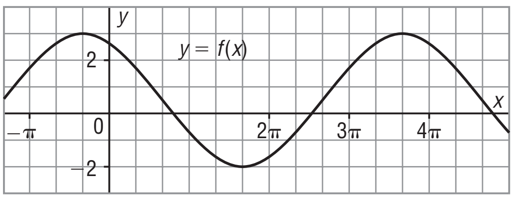


Sample response: I graphed $y = \frac{1}{2} \sin 3x$, shifted several points $\frac{\pi}{6}$ units left and 2 units up, then joined the points to get the graph of $y = \frac{1}{2} \sin 3\left(x + \frac{\pi}{6}\right) + 2$.

b) List the characteristics of the graph you drew.

The amplitude is $\frac{1}{2}$. The period is $\frac{2\pi}{3}$. There are no zeros.
The domain is $-2\pi \leq x \leq 2\pi$. The range is $\frac{3}{2} \leq y \leq \frac{5}{2}$.

12. An equation of the function graphed below has the form $y = a \cos b(x - c) + d$. Identify the values of a, b, c, and d in the equation, then write an equation for the function.



Sample response: The equation of the centre line is $y = \frac{1}{2}$, so the vertical translation is $\frac{1}{2}$ unit up and $d = \frac{1}{2}$.

The amplitude is: $\frac{3 - (-2)}{2} = \frac{5}{2}$, so $a = \frac{5}{2}$

Choose the x-coordinates of two adjacent maximum points, $-\frac{\pi}{3}$ and $\frac{11\pi}{3}$. The period is: $\frac{11\pi}{3} - \left(-\frac{\pi}{3}\right) = 4\pi$

So, b is: $\frac{2\pi}{4\pi} = \frac{1}{2}$

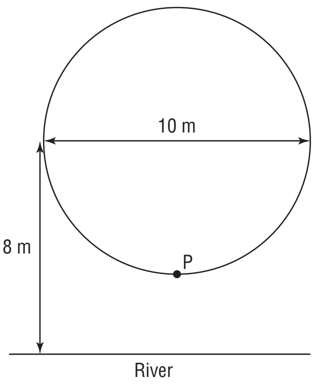
To the left of the y-axis, the cosine function begins its cycle at $x = -\frac{\pi}{3}$, so a possible phase shift is $-\frac{\pi}{3}$, and $c = -\frac{\pi}{3}$.

Substitute for a, b, c, and d in: $y = a \cos b(x - c) + d$

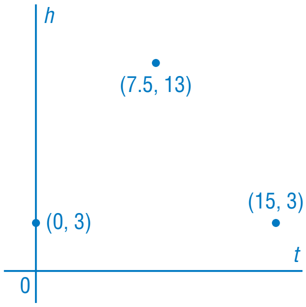
An equation is: $y = \frac{5}{2} \cos \frac{1}{2}\left(x + \frac{\pi}{3}\right) + \frac{1}{2}$

6.7

13. A water wheel has diameter 10 m and completes 4 revolutions each minute. The axle of the wheel is 8 m above a river.



a) The wheel is at rest at time $t = 0$ s, with point P at the lowest point on the wheel. Determine a function that models the height of P above the river, h metres, at any time t seconds. Explain how the characteristics of the graph relate to the given information.



The time for 1 revolution is 15 s.
At $t = 0$, $h = 3$
At $t = 7.5$, $h = 13$
The graph begins at (0, 3), which is a minimum point.
The first maximum point is at (7.5, 13).
The next minimum point is after 1 cycle and it has coordinates (15, 3).
The position of the first maximum is known, so use a cosine function:
$h(t) = a \cos b(t - c) + d$
The constant in the equation of the centre line of the graph is the height of the axle above the river, so its equation is: $h = 8$; and this is also the vertical translation, so $d = 8$
The amplitude is one-half the diameter of the wheel, so $a = 5$
The period is the time for 1 revolution, so $b = \frac{2\pi}{15}$
A possible phase shift is: $c = 7.5$
An equation is: $h(t) = 5 \cos \frac{2\pi}{15}(t - 7.5) + 8$

b) Use technology to graph the function. Use this graph to determine:

i) the height of P after 35 s

Graph: $Y = 5 \cos \frac{2\pi}{15}(X - 7.5) + 8$
Determine the Y-value when $X = 35$.
After 35 s, P is 10.5 m high.

ii) the times, to the nearest tenth of a second, in the first 15 s of motion that P is 11 m above the river

Graph: $Y = 5 \cos \frac{2\pi}{15}(X - 7.5) + 8$ and $Y = 11$

Determine the Y-coordinates of the first two points of intersection.

P is 11 m above the river after approximately 5.3 s and 9.7 s.