Lesson 5.4 Exercises, pages 381–385

A

4. Evaluate each logarithm.

a) $\log_4 16$

$= \log_4 4^2$
$= 2$

b) $\log 100\,000$

$= \log_{10} 10^5$
$= 5$

c) $\log_6 1296$

$= \log_6 6^4$
$= 4$

d) $\log_2 2$

$= 1$

5. Write each exponential expression as a logarithmic expression.

a) $2^6 = 64$

The base is 2.
The logarithm is 6.
So, $6 = \log_2 64$

b) $10^4 = 10\,000$

The base is 10.
The logarithm is 4.
So, $4 = \log 10\,000$

c) $4^{-\frac{1}{2}} = \frac{1}{2}$

The base is 4.
The logarithm is $-\frac{1}{2}$.
So, $-\frac{1}{2} = \log_4\left(\frac{1}{2}\right)$

d) $3^{\frac{2}{3}} = \sqrt[3]{9}$

The base is 3.
The logarithm is $\frac{2}{3}$.
So, $\frac{2}{3} = \log_3 \sqrt[3]{9}$

6. a) Write each logarithmic expression as an exponential expression.

i) $\log_7 16\,807 = 5$

The base is 7.
The exponent is 5.
So, $16\,807 = 7^5$

ii) $\log_9 3 = \frac{1}{2}$

The base is 9.
The exponent is $\frac{1}{2}$.
So, $3 = 9^{\frac{1}{2}}$

iii) $\log 0.01 = -2$

The base is 10.
The exponent is -2.
So, $0.01 = 10^{-2}$

iv) $\log_3\left(\frac{\sqrt{3}}{3}\right) = -\frac{1}{2}$

The base is 3.
The exponent is $-\frac{1}{2}$.
So, $\frac{\sqrt{3}}{3} = 3^{-\frac{1}{2}}$

b) Use one pair of statements from part a to explain the relationship between a logarithmic expression and an exponential expression.

Sample response: I start with this logarithmic expression: $\log_7 16\,807 = 5$. The logarithm of a number is the power to which the base of the logarithm is raised to get the number. So, the logarithm base 7 of 16 807 is the power to which I raise 7 to get 16 807, which is 5. That is, $16\,807 = 7^5$, which is the equivalent exponential expression.

B

7. Evaluate each logarithm.

a) $\log_4 8$

$= \log_4(2^3)$
$= \log_4\left(4^{\frac{1}{2}}\right)^3$
$= \log_4 4^{\frac{3}{2}}$
$= \frac{3}{2}$

b) $\log_9\left(\frac{1}{27}\right)$

$= \log_9 3^{-3}$
$= \log_9\left(9^{\frac{1}{2}}\right)^{-3}$
$= \log_9\left(9^{-\frac{3}{2}}\right)$
$= -\frac{3}{2}$

c) $\log_6 1$

$= \log_6 6^0$
$= 0$

d) $\log_2\left(4\sqrt[4]{2}\right)$

$= \log_2\left(2^2 \cdot 2^{\frac{1}{4}}\right)$
$= \log_2\left(2^{\frac{9}{4}}\right)$
$= \frac{9}{4}$

8. a) Write 3 as a logarithm with base 2.

Use: $n = \log_b b^n$
$3 = \log_2 2^3$
So, $3 = \log_2 8$

b) Write 2 as a logarithm with base 3.

Use: $n = \log_b b^n$
$2 = \log_3 3^2$
So, $2 = \log_3 9$

9. Determine the value of $\log_b 1$. Justify the answer.

$\log_b 1$ is the exponent of base b, when b is raised to a power and the result is 1. The only way to get 1 is to raise the base to the power 0. So, $\log_b 1 = 0$

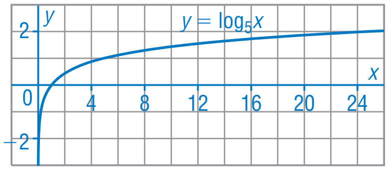
10. How are the domain and range of the functions $y = b^x$ and $y = \log_b x$ related?

Since the functions $y = b^x$ and $y = \log_b x$ are inverse functions, the domain of $y = b^x$ is the range of $y = \log_b x$, and the range of $y = b^x$ is the domain of $y = \log_b x$.

11. **a)** Use a table of values to graph $y = \log_5 x$.

Determine values for $y = 5^x$, then interchange the coordinates for the table of values for $y = \log_5 x$.

x	$y = \log_5 x$
$\frac{1}{5}$	-1
1	0
5	1
25	2



b) Identify the intercepts, the equation of the asymptote, the domain, and the range of the function.

The graph does not intersect the y-axis, so it does not have a y-intercept.
The graph has x-intercept 1.
The y-axis is a vertical asymptote; its equation is $x = 0$.
The domain of the function is $x > 0$.
The range of the function is $y \in \mathbb{R}$.

c) What is the significance of the asymptote?

The asymptote signifies that the logarithm of any positive number very close to 0 exists, but the logarithm of 0 does not exist. The graph approaches the line $x = 0$ but never intersects it.

12. **a)** Use technology to graph $y = \log x$. Identify the intercepts, the equation of the asymptote, the domain, and the range of the function.

On the Y= screen, input $Y_1 = \log(X)$, then press: $\boxed{\text{GRAPH}}$.
From the table of values, or the CALC feature, there is no y-intercept, the x-intercept is 1, and the equation of the asymptote is $x = 0$.
The domain is $x > 0$; the range is $y \in \mathbb{R}$.

b) What is the equation of the inverse of $y = \log x$?

$y = \log x$ can be written $y = \log_{10} x$.
Interchange x and y, then solve for y.
$x = \log_{10} y$ Write the equivalent exponential statement.
$y = 10^x$

13. Use benchmarks to estimate the value of each logarithm, to the nearest tenth.

a) $\log_3 12$

Identify powers of 3 close to 12.
$3^2 = 9$ and $3^3 = 27$
$\log_3 3^2 < \log_3 12 < \log_3 3^3$
So, $2 < \log_3 12 < 3$
An estimate is: $\log_3 12 \doteq 2.2$
Check.
$3^{2.2} \doteq 11.21157846$
$3^{2.3} \doteq 12.51350253$
So, $\log_3 12 \doteq 2.3$

b) $\log_2 100$

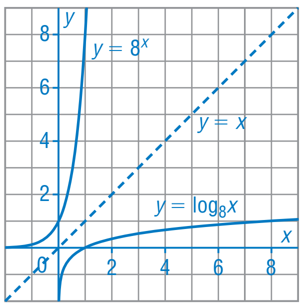
Identify powers of 2 close to 100.
$2^6 = 64$ and $2^7 = 128$
$\log_2 2^6 < \log_2 100 < \log_2 2^7$
So, $6 < \log_2 100 < 7$
An estimate is: $\log_2 100 \doteq 6.6$
Check.
$2^{6.6} \doteq 97.00586026$
$2^{6.7} \doteq 103.9683067$
So, $\log_2 100 \doteq 6.6$

14. Write the equations of an exponential function and a logarithmic function with the same base. Use graphs of these functions to demonstrate that each function is the inverse of the other.

Sample response: Here are the tables of values and graphs of $y = 8^x$ and $y = \log_8 x$. Plot the points, then join them with smooth curves.

x	$y = 8^x$
-1	0.125
0	1
1	8

x	$y = \log_8 x$
0.125	-1
1	0
8	1



From the table, the functions are inverses because the coordinates of corresponding points are interchanged.
From the graph, the functions are inverses because their graphs are reflections of each other in the line $y = x$.

15. Use benchmarks to estimate the value of each logarithm to the nearest tenth.

a) $\log_2 6.5$

Identify powers of 2 close to 6.5.
$2^2 = 4$ and $2^3 = 8$
So, $2 < \log_2 6.5 < 3$
An estimate is: $\log_2 6.5 \doteq 2.7$
Check.
$2^{2.7} \doteq 6.498019171$
$2^{2.8} \doteq 6.964404506$
So, $\log_2 6.5 \doteq 2.7$

b) $\log_3 1.8$

Identify powers of 3 close to 1.8.
$3^0 = 1$ and $3^1 = 3$
So, $0 < \log_3 1.8 < 1$
An estimate is:
$\log_3 1.8 \doteq 0.6$
Check.
$3^{0.6} \doteq 1.933182045$
$3^{0.5} \doteq 1.732050808$
So, $\log_3 1.8 \doteq 0.5$

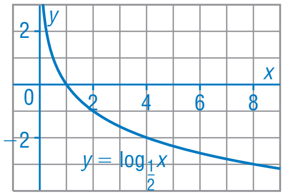
C

16. Graph $y = \log_{\frac{1}{2}}x$. How is the graph of this function related to the graph of $y = \log_2 x$?

Make a table of values for $y = \left(\frac{1}{2}\right)^x$, then interchange the coordinates for $y = \log_{\frac{1}{2}}x$. Plot the points, then join them with a smooth curve.

x	$y = \left(\frac{1}{2}\right)^x$
−3	8
−2	4
−1	2
0	1
1	0.5
2	0.25

x	$y = \log_{\frac{1}{2}}x$
8	−3
4	−2
2	−1
1	0
0.5	1
0.25	2

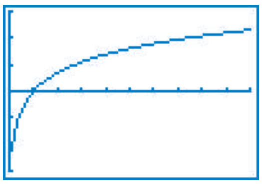


I looked at the graph of $y = \log_2 x$ that I drew on page 375.
The graph of $y = \log_{\frac{1}{2}}x$ is the reflection of the graph of $y = \log_2 x$ in the x-axis.

17. On a graphing calculator, the key [LN] calculates the value of a logarithm whose base is the irrational number e. The number e is known as Euler's constant. Logarithms with base e are called *natural logarithms.*

a) Graph $y = \ln x$. Sketch the graph.

Input: Y_1 = [LN] (X), then press: [GRAPH]



b) Determine the value of e to the nearest thousandth.

When $y = 1$, $\log_e x = 1$, so $x = e^1$, or $x = e$
On the screen in part a, graph $Y_2 = 1$, then determine the approximate x-coordinate of the point of intersection: 2.7182818
The value of e is approximately 2.718.

