Lesson 8.4 Exercises, pages 727–732

A

4. Evaluate.

a) $\frac{10!}{3!7!}$

$= \frac{\cancel{10}^5 \cdot \cancel{9}^3 \cdot 8}{\cancel{3} \cdot \cancel{2} \cdot 1}$

$= 120$

b) $\frac{6!}{1!5!}$

$= \frac{6}{1!}$

$= 6$

c) $\frac{12!}{3!9!}$

$= \frac{\cancel{12}^4 \cdot 11 \cdot \cancel{10}^5}{\cancel{3} \cdot \cancel{2} \cdot 1}$

$= 220$

d) $\frac{15!}{13!2!}$

$= \frac{15 \cdot \cancel{14}^7}{\cancel{2}}$

$= 105$

5. Determine each value.

a) ${}_4C_2$

$= \frac{4!}{(4 - 2)!2!}$

$= \frac{4!}{2!2!}$

$= \frac{\cancel{4}^2 \cdot 3}{\cancel{2}}$

$= 6$

b) $\binom{7}{5}$

$= \frac{7!}{(7 - 5)!5!}$

$= \frac{7!}{2!5!}$

$= \frac{7 \cdot \cancel{6}^3}{\cancel{2}}$

$= 21$

c) ${}_{10}C_{10}$

$= \frac{10!}{(10 - 10)!10!}$

$= \frac{10!}{0!10!}$

$= 1$

d) $\binom{12}{8}$

$= \frac{12!}{(12 - 8)!8!}$

$= \frac{12!}{4!8!}$

$= \frac{\cancel{12}^3 \cdot 11 \cdot \cancel{10}^5 \cdot \cancel{9}^3}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$= 495$

6. How many combinations of each number of letters can be formed from the letters in the word LINE? List the combinations each time.

a) 1

${}_4C_1 = \frac{4!}{(4 - 1)!1!}$

$= \frac{4!}{3!1!}$

$= 4$

They are: L, I, N, E

b) 2

${}_4C_2 = \frac{4!}{(4 - 2)!2!}$

$= \frac{4!}{2!2!}$

$= \frac{\cancel{4}^2 \cdot 3}{\cancel{2}}$

$= 6$

They are: LI, LN, LE, IN, IE, NE

c) 3

$$ {}_4C_3 = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = 4 $$

They are: LIN, LIE, LNE, INE

d) 4

$$ {}_4C_4 = \frac{4!}{(4-4)!4!} = \frac{4!}{0!4!} = 1 $$

It is: LINE

B

7. These are the names of lakes in western Canada. How many 4-letter combinations can be formed using the letters in each name?

a) BISTCHO

There are 7 letters.

$$ {}_7C_4 = \frac{7!}{(7-4)!4!} = \frac{7!}{3!4!} = \frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3} \cdot \cancel{2} \cdot 1} = 35 $$

35 combinations are possible.

b) TOEWS

There are 5 letters.

$$ {}_5C_4 = \frac{5!}{(5-4)!4!} = \frac{5!}{1!4!} = 5 $$

5 combinations are possible.

c) HOIDAS

There are 6 letters.

$$ {}_6C_4 = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{{}^3\cancel{6} \cdot 5}{\cancel{2}} = 15 $$

15 combinations are possible.

d) COQUITLAM

There are 9 letters.

$$ {}_9C_4 = \frac{9!}{(9-4)!4!} = \frac{9!}{5!4!} = \frac{9 \cdot \cancel{8} \cdot 7 \cdot {}^2\cancel{6}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 126 $$

126 combinations are possible.

8. a) How many 3-digit permutations are there of the digits in the number 67 512?

There are 5 digits. All digits are different.

Number of 3-digit permutations is: $ {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60 $

b) How can you use your answer to part a to determine how many 3-digit combinations are possible?

Order does not matter: $ {}_nC_r = \frac{{}_nP_r}{r!} $

So, the number of 3-digit combinations is: $ {}_5C_3 = \frac{{}_5P_3}{3!} = \frac{60}{6} = 10 $

9. Would you use a permutation or a combination to represent each situation? Justify your choice.

a) choosing 3 out of 4 musical notes to create a tune

Order matters because different orders of musical notes create different tunes. I would use a permutation.

b) choosing 3 out of 4 sweatshirts to take camping

It doesn't matter in which order I choose the sweatshirts. I would use a combination.

c) choosing 3 out of 4 contestants to advance to the next round

It doesn't matter in which order the contestants advance to the next round. I would use a combination.

d) choosing 3 out of 4 digits to create a password

Order matters because different orders of digits create different passwords. I would use a permutation.

10. Rafael has a list of his mom's 15 favourite songs. He will download 7 of these songs to her iPod.

a) How many ways can Rafael select 7 songs to download?

Order does not matter.

$$\begin{aligned} {}_{15}C_7 &= \frac{15!}{(15 - 7)!7!} \\ &= \frac{15!}{8!7!} \\ &= 6435 \end{aligned}$$

Rafael can select 7 songs to download in 6435 ways.

b) Suppose Rafael downloads 8 songs. Without doing any calculations, how many ways can he select 8 songs? Explain your strategy.

The number of ways of downloading 8 songs from 15 songs is the same as the number of ways of downloading 7 songs (that is, not downloading 8 songs) from 15 songs. So, Rafael can select 8 songs to download in 6435 ways.

11. At the Soccer World Cup, 16 of the 32 teams advance beyond the second round. How many ways can 16 teams advance? Did you use a permutation or a combination to solve this problem? Explain.

The order in which the teams advance does not matter so I will use a combination.

$$\begin{aligned} {}_{32}C_{16} &= \frac{32!}{(32 - 16)!16!} \\ &= \frac{32!}{16!16!} \\ &= 601\,080\,390 \end{aligned}$$

The teams can advance in 601 080 390 ways.

12. When Tanner's team won the final game in the Genesis Hospitality High School hockey tournament in Brandon, Manitoba, each of the 6 players on the ice gave each other a high five. How many high fives were there?

The order in which the players give high fives does not matter so I will use a combination to choose pairs of players.

$$\begin{aligned} {}_{6}C_{2} &= \frac{6!}{(6 - 2)!2!} \\ &= \frac{6!}{4!2!} \\ &= 15 \end{aligned}$$

There were 15 high fives.

13. A test has 2 parts. Students must answer 10 of 15 questions from part A and write 3 essays from a choice of 5 essay topics in part B. What is the number of possible responses to the test?

Number of ways of choosing 10 questions from part A:

$$\begin{aligned} {}_{15}C_{10} &= \frac{15!}{(15 - 10)!10!} \\ &= \frac{15!}{5!10!} \\ &= 3003 \end{aligned}$$

Number of ways of choosing 3 essays in part B:

$$\begin{aligned} {}_{5}C_{3} &= \frac{5!}{(5 - 3)!3!} \\ &= \frac{5!}{2!3!} \\ &= 10 \end{aligned}$$

Then use the counting principle: $3003 \cdot 10 = 30\,030$

There are 30 030 possible responses.

14. A jury of 6 men and 6 women is to be chosen from a jury pool of 12 men and 15 women. How many juries are possible?

Number of ways of choosing men: $_{12}C_6 = \frac{12!}{(12 - 6)!6!}$

$= \frac{12!}{6!6!}$

$= 924$

Number of ways of choosing women: $_{15}C_6 = \frac{15!}{(15 - 6)!6!}$

$= \frac{15!}{9!6!}$

$= 5005$

Then use the fundamental counting principle: $5005 \cdot 924 = 4\,624\,620$

There are 4 624 620 possible juries.

15. Solve each equation for n or r.

a) $_nC_2 = 28$

$_nC_2 = \frac{n!}{(n - 2)!2!}$

$28 = \frac{n!}{(n - 2)!2}$

$2 \cdot 28 = n(n - 1)$

$0 = n^2 - n - 56$

$0 = (n - 8)(n + 7)$

$n = 8$ or $n = -7$

Since n cannot be negative,

$n = 8$

b) $_nC_4 = 35$

$_nC_4 = \frac{n!}{(n - 4)!4!}$

$35 = \frac{n!}{(n - 4)!24}$

$24 \cdot 35 = n(n - 1)(n - 2)(n - 3)$

$840 = n(n - 1)(n - 2)(n - 3)$

Use a calculator.

$\sqrt[4]{840} \doteq 5.4$

So, try 4 consecutive numbers

with 5 as one of the middle numbers:

$7 \cdot 6 \cdot 5 \cdot 4 = 840$

So, $n = 7$

c) $_4C_r = 6$

$_4C_r = \frac{4!}{(4 - r)!r!}$

$6 = \frac{24}{(4 - r)!r!}$

$(4 - r)!r! = \frac{24}{6}$

$(4 - r)!r! = 4$

Use guess and test.

Since $2! = 2$

$(4 - 2)!2! = 2 \cdot 2$

$= 4$

So, $r = 2$

d) $_6C_r = 20$

$_6C_r = \frac{6!}{(6 - r)!r!}$

$20 = \frac{720}{(6 - r)!r!}$

$(6 - r)!r! = \frac{720}{20}$

$(6 - r)!r! = 36$

Use guess and test.

Since $3! = 6$

$(6 - 3)!3! = 6 \cdot 6$

$= 36$

So, $r = 3$

C

16. From a standard deck of 52 playing cards, how many ways can each hand of 5 cards be dealt?

a) any 5 cards

Order does not matter.

$$_{52}C_5 = \frac{52!}{(52 - 5)!5!}$$
$$= \frac{52!}{47!5!}$$
$$= 2\ 598\ 960$$

So, a hand of any 5 cards can be dealt in 2 598 960 ways.

b) 5 black cards

There are 26 black cards in a deck of 52 cards.
The number of combinations of 5 cards drawn from 26 is:

$$_{26}C_5 = \frac{26!}{(26 - 5)!5!}$$
$$= 65\ 780$$

So, a hand of 5 black cards can be dealt in 65 780 ways.

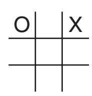
c) exactly 2 diamonds

There are 13 diamonds and 39 not diamonds in a deck of 52 playing cards. So, there are $_{13}C_2$ ways to deal exactly 2 diamonds and $_{39}C_3$ ways to deal 3 not diamonds. By the fundamental counting principle, the total number of ways to deal this type of hand is:

$$_{13}C_2 \cdot {}_{39}C_3 = 78 \cdot 9139$$
$$= 712\ 842$$

So, a hand of exactly 2 diamonds can be dealt in 712 842 ways.

17. Two players take turns writing X and O in a 3-by-3 grid until all the cells are full. How many ways are there to fill all the cells with Xs and Os?



The order in which the Xs and Os are written does not matter, so use combinations.
If X starts the game, there will be 5 Xs and 4 Os.
The number of ways to write 5 Xs in the cells is:

$$_9C_5 = \frac{9!}{4!5!}$$
$$= 126$$

Then fill up the rest of the cells with Os.

If O starts the game, there will be 5 Os and 4 Xs.
The number of ways to write 5 Os in the cells is also 126.
Then fill up the rest of the cells with Xs.

So, the number of ways to fill all the cells with Xs and Os is:
$126 + 126 = 252$
The cells can be filled in 252 ways.