Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2013

Manitoba

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.

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Disponible en français.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
Booklet 1*	9 Short-Answer	13	31
	6 Long-Answer	18	
Booklet 2	9 Multiple-Choice	9	58
	15 Short-Answer	17	
	10 Long-Answer	32	
Total			**89**

* The first 6 questions in *Booklet 1* require a calculator. 
You have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = Pe^{rt}$$

$$\log_a(MN) = \log_a M + \log_a N$$
$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$
$$\log_a(M^n) = n\log_a M$$
$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$
$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$y = a\sin b(x - c) + d$$
$$y = a\cos b(x - c) + d$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are explained below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Solve: Give a solution for a problem or determine the value(s) of a variable.

Verify: Establish the truth of a statement by substitution or comparison.

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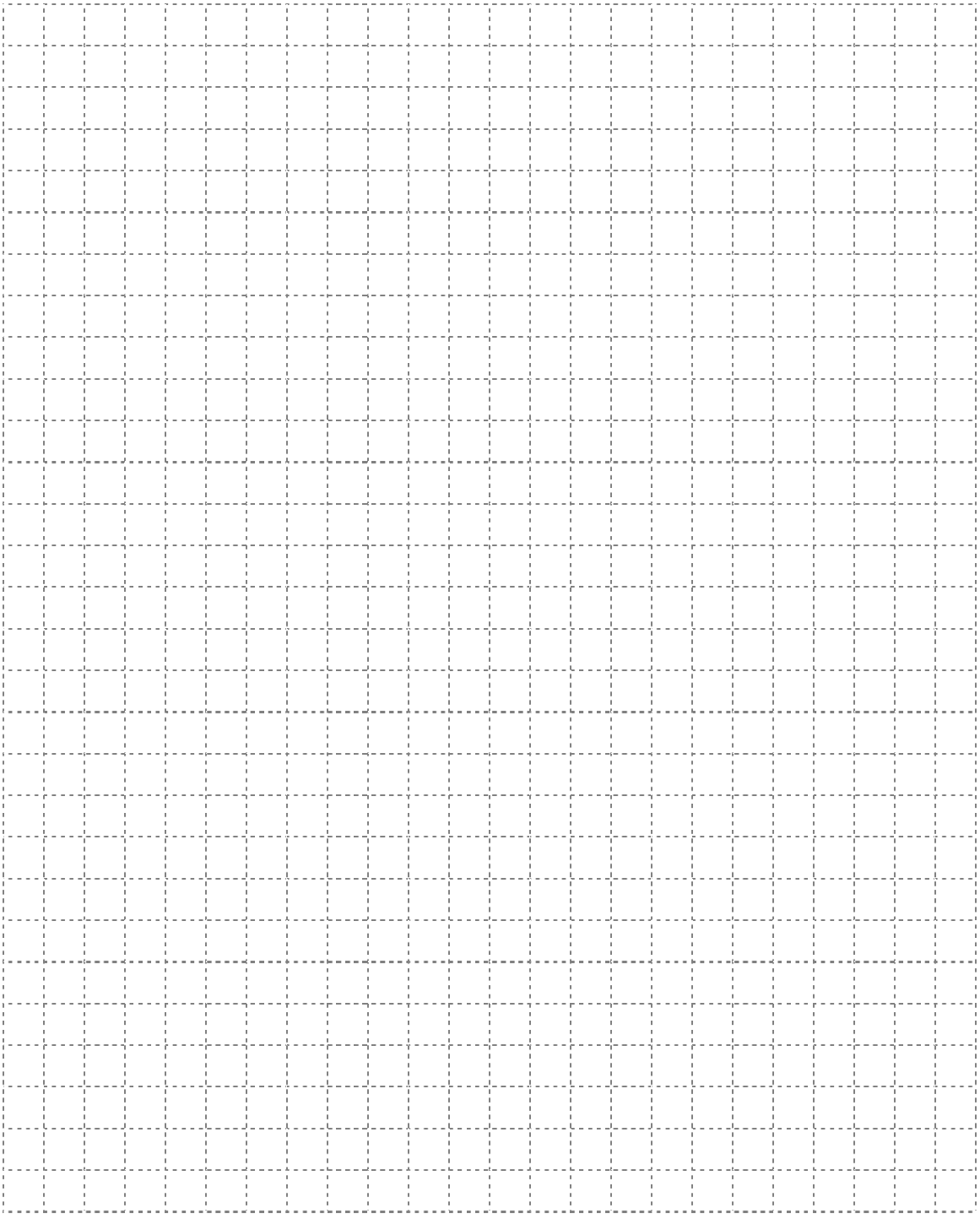


Instructions

- There are 15 questions for a total of 31 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

No marks will be awarded for work done on this page.

Question 1  3 marks

101

Gina correctly started to answer the following question. Complete her solution.

Question: Solve the following equation for all real values of θ.
Express your answer in radians correct to 3 decimal places.

$$3\sin^2\theta - 14\sin\theta - 5 = 0$$

Gina's solution: $3\sin^2\theta - 14\sin\theta - 5 = 0$

$$(3\sin\theta + 1)(\sin\theta - 5) = 0$$

Question 2  3 marks

102

Find and simplify the 6th term in the binomial expansion of $\left(3x^4 - \frac{1}{x^3}\right)^9$.

Question 3  3 marks

The number of times a website is visited can be modeled by the function:

$$A = 800(e)^{rt}$$

where $A =$ the total number of visitors at time t

$t =$ the time in days $(t \geq 0)$

$r =$ the rate of growth

After 5 days, 40 000 people have visited the site.
Determine the number of visitors expected after 9 days.

Express your answer as a whole number.

Question 4  3 marks 104

Solve algebraically:

$$10^{3x} = 7^{x+5}$$

Express your answer correct to 3 decimal places.

Question 5  2 marks

105

A word contains two Ms, two Es, two Ns, and no other repeated letters.

Suppose one of the Ns is replaced by an M.
Will this replacement result in greater or fewer permutations?

Justify your reasoning.

Question 6  3 marks

106

There is a group of 16 boys and 12 girls. How many ways can a committee of 3 people be formed if there must be at least 2 girls on the committee?

Express your answer as a whole number.

Note: A calculator is not required for the remaining test questions.

Question 7 — 2 marks — 107

A student is using the formula $s = \theta r$ to find an arc length of a circle. Given a central angle measure of 35° and a radius of 6 cm, the student's solution is as follows:

$s = (35)(6)$

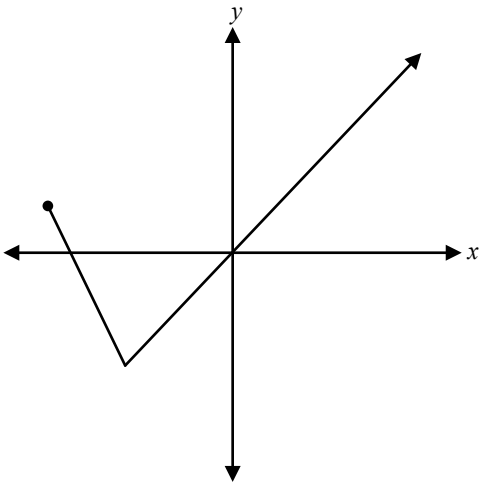
$s = 210 \text{ cm}$

Explain why this solution is incorrect.

Write the correct solution.

Question 8 — 1 mark — 108

Given the graph of $f(x)$ below, explain how you would sketch the graph of $y = |f(x)|$.



Question 9 — 1 mark — 109

Claire correctly solves the following equation:

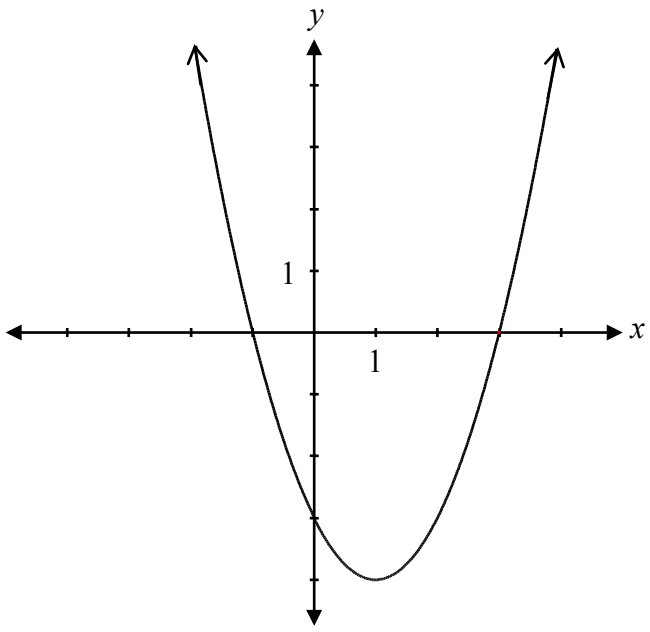
$$\log_2(6-x)+\log_2(3-x)=2$$

She finds two possible values of x: $x=2$ and $x=7$.

Identify which one of these values is unacceptable and explain why.

Question 10 — 1 mark — 110

Given the graph of the function $f(x)$ below, state the domain of $y=\sqrt{f(x)}$.



Question 11 — 1 mark — 111

A school offers 4 different Science courses, 3 different Mathematics courses, and 2 different English courses.

Julie must select 1 Science course, 1 Mathematics course, and 1 English course. She thinks this creates 9 options for her timetable.

Show why Julie is incorrect.

Question 12 — 1 mark — 112

Explain how Pascal's triangle can be used to determine the coefficients in the binomial expansion of $(x + y)^n$.

Question 13 — 3 marks

113

Prove the identity below for all permissible values of x:

$$\frac{1+\cos 2x}{\sin 2x} = \cot x$$

Left-Hand Side	Right-Hand Side

Question 14 — 2 marks — 114

Your classmate, Leo, was absent for one of his math lessons.

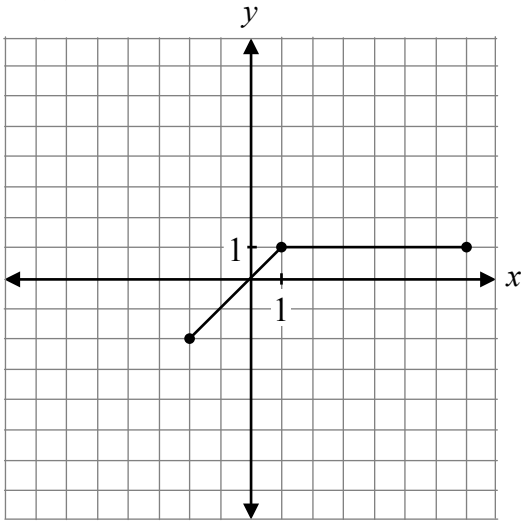
Explain to Leo how to determine the cosecant ratio for an angle in standard position given that $P(-3, -4)$ is a point on the terminal arm of the angle.

Question 15 — 2 marks

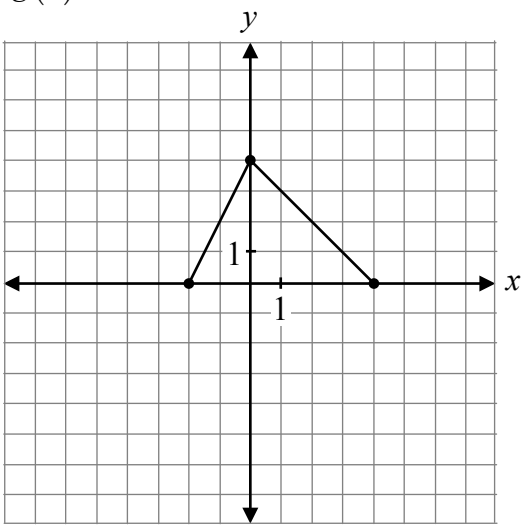
115

Given the following graphs:

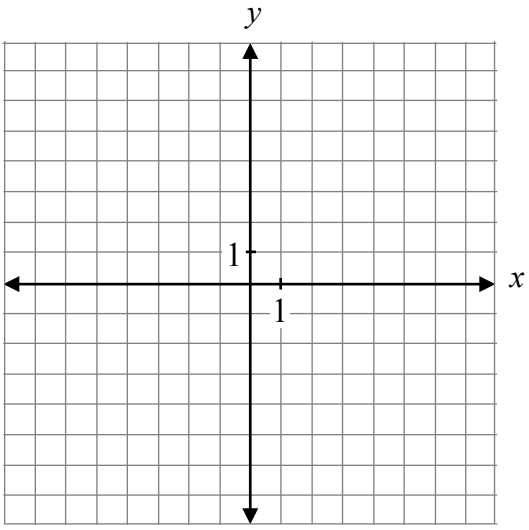
$f(x)$



$g(x)$



Sketch the graph of $f(x) + g(x)$.



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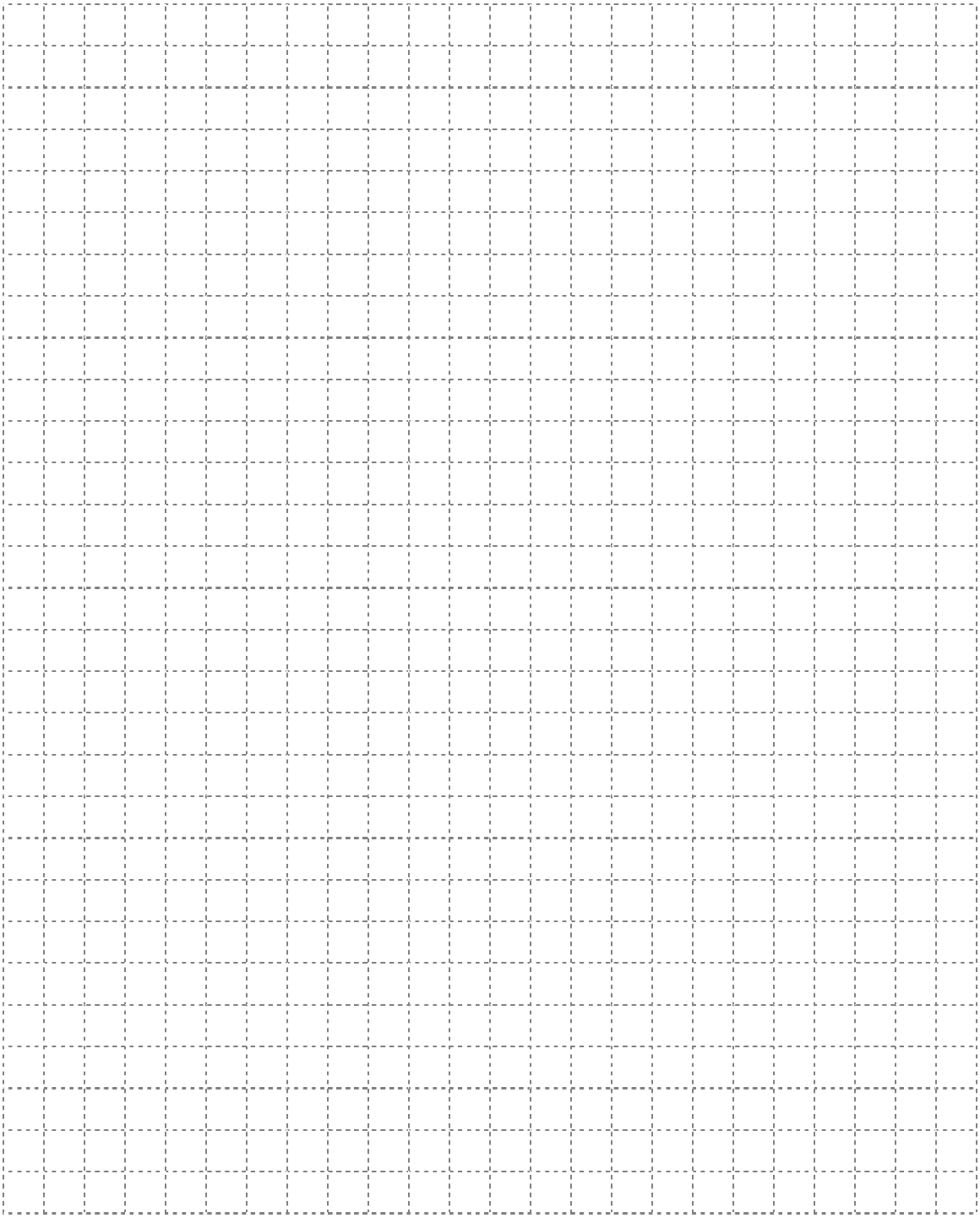

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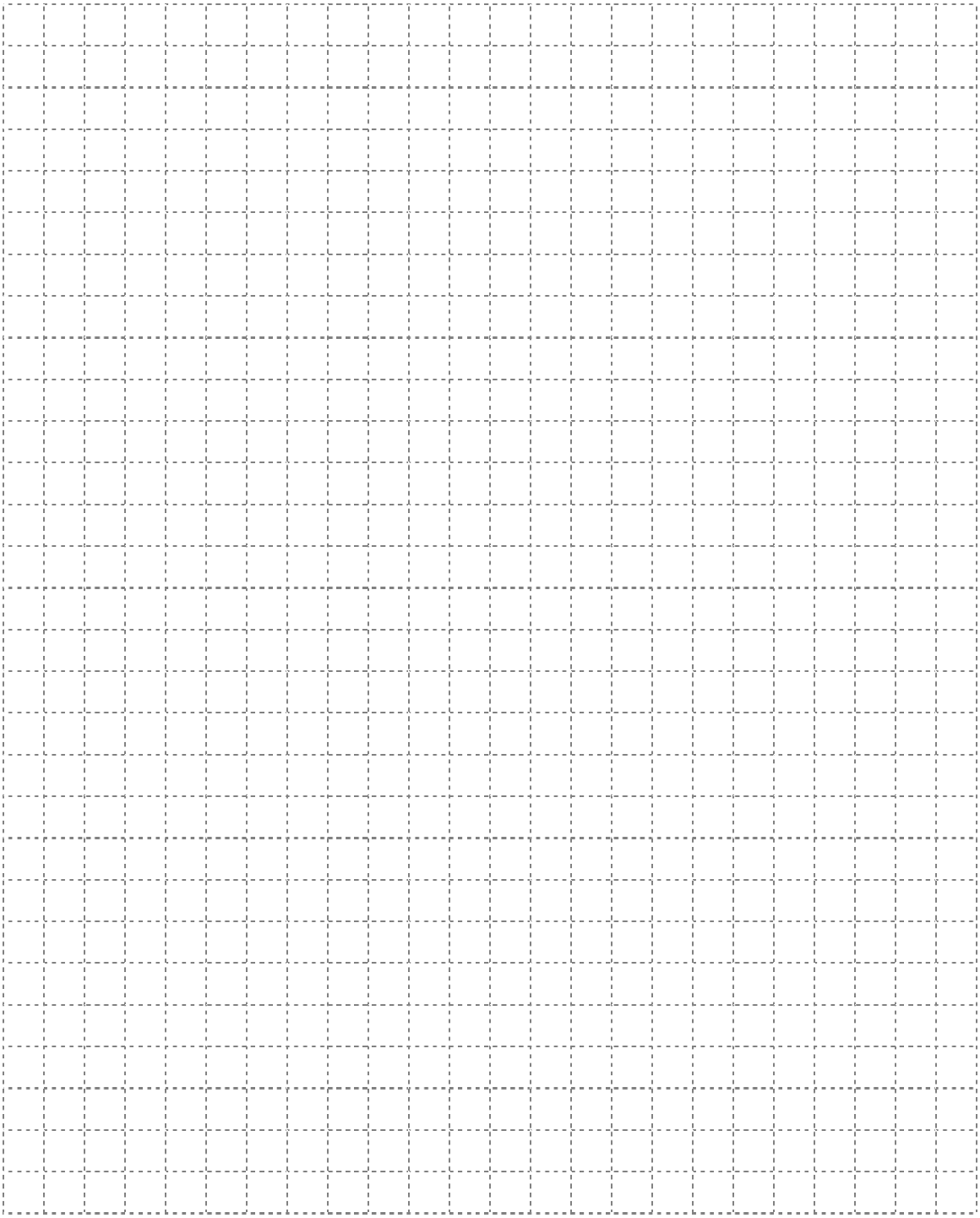

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