Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2013

Manitoba

Manitoba Education Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.
Booklet 1. June 2013 [electronic resource]

ISBN: 978-0-7711-5432-4

1. Mathematics—Examinations, questions, etc.
2. Mathematics—Examinations.
3. Educational tests and measurements—Manitoba.
4. Mathematics—Study and teaching (Secondary)—Manitoba.
5. Calculus—Study and teaching (Secondary)—Manitoba.
6. Mathematical ability—Testing.
I. Manitoba. Manitoba Education.
515.76

Manitoba Education
School Programs Division
Winnipeg, Manitoba, Canada

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.

This resource will also be available on the Manitoba Education website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
Booklet 1*	8 Short-Answer	12	38
	8 Long-Answer	26	
Booklet 2	8 Multiple-Choice	8	52
	16 Short-Answer	27	
	5 Long-Answer	17	
Total			**90**

* The first 4 questions in *Booklet 1* require a calculator. 
You have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = Pe^{rt}$$

$$\log_a(MN) = \log_a M + \log_a N$$
$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$
$$\log_a(M^n) = n\log_a M$$
$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$
$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$y = a\sin b(x - c) + d$$
$$y = a\cos b(x - c) + d$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are explained below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Solve: Give a solution for a problem or determine the value(s) of a variable.

Verify: Establish the truth of a statement by substitution or comparison.

Scrap Paper

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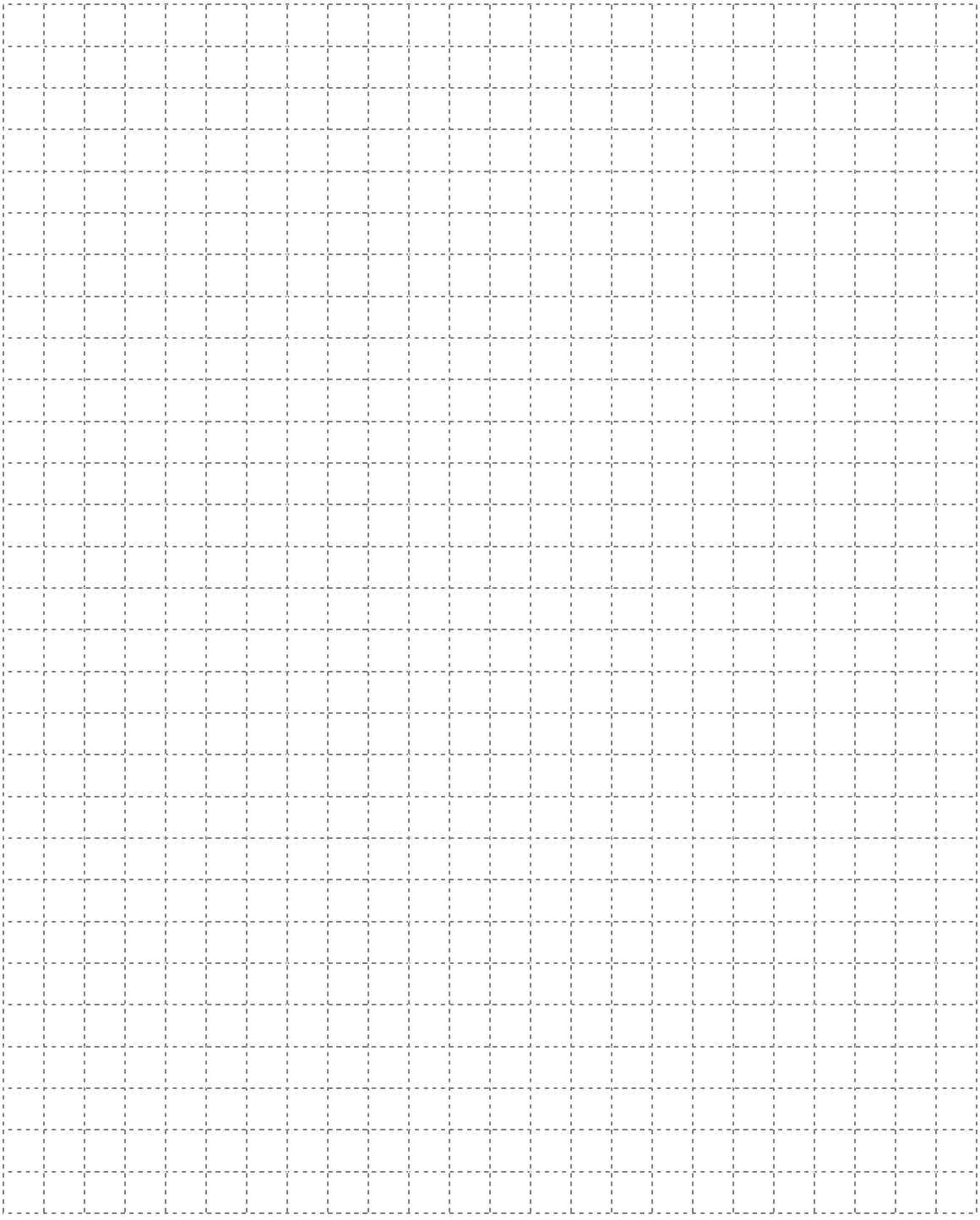


Instructions

- There are 16 questions for a total of 38 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

No marks will be awarded for work done on this page.

Question 1  2 marks

101

A central angle of a circle subtends an arc length of 5π cm.
Given the circle has a radius of 9 cm, find the measure of the central angle in degrees.

Question 2  4 marks

Solve the equation $\csc^2\theta + 3\csc\theta - 4 = 0$ over the interval $[0, 2\pi]$.
Express your answers as exact values or correct to 3 decimal places.

Question 3  3 marks

103

Jess invests $12 000 at a rate of 4.75% compounded monthly.
How long will it take for Jess to triple her investment?

Express your answer in years, correct to 3 decimal places.

Question 4  3 marks

104

The 4th term in the binomial expansion of $\left(qx^2 - \frac{3}{x}\right)^{10}$ is $414\ 720x^{11}$.

Determine the value of q algebraically.

Note: A calculator is not required for the remaining test questions.

Question 5 — 1 mark — 105

Bella has 2 pairs of shoes, 3 pairs of pants, and 10 shirts.
Carey has 4 pairs of shoes, 4 pairs of pants, and 4 shirts.
An outfit is made up of one pair of shoes, one pair of pants, and one shirt.

Who can make more outfits? Justify your answer.

Question 6 — 2 marks — 106

In the binomial expansion of $(x - y)^{10}$, how many terms will be positive?

Justify your answer.

Question 7 — 4 marks — 107

Solve the following equation algebraically where $180^\circ \leq \theta \leq 360^\circ$.

$$2\sin^2\theta + 5\cos\theta + 1 = 0$$

Question 8 — 3 marks — 108

Solve the following equation algebraically:

$$\log_3(x-4) + \log_3(x-2) = 1$$

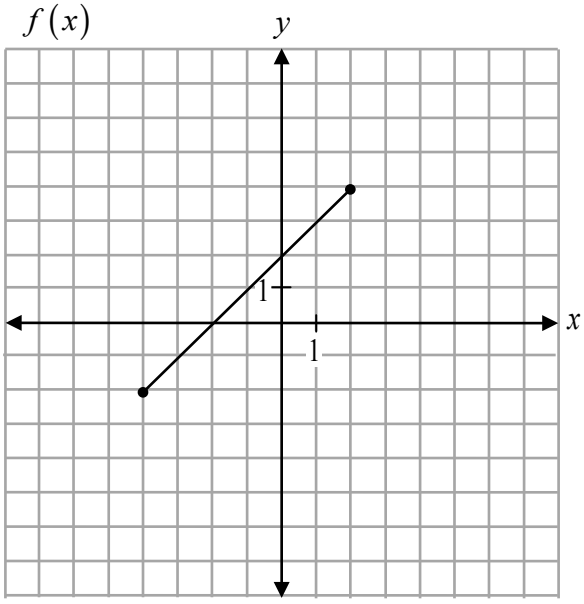
Question 9 — 1 mark — 109

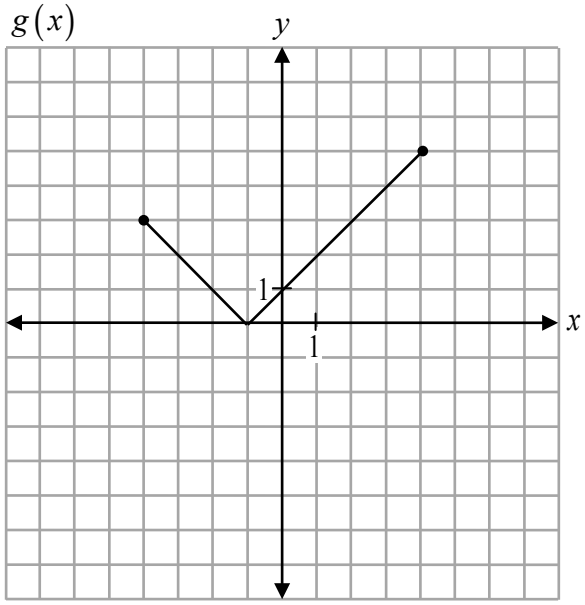
Given that $f(x) = \{(1, 3), (2, 5), (3, 4), (4, 2)\}$, find $f(f(3))$.

Question 10 — 2 marks

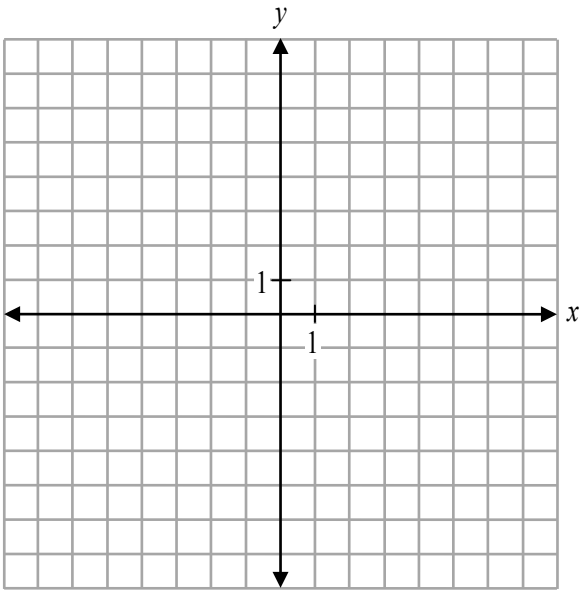
110

Given the graphs of $f(x)$ and $g(x)$ below,





sketch the graph of $y = f(x) - g(x)$.



Question 11 — 2 marks — 111

Given the graph of $y = f(x)$, describe the transformations to obtain the graph of the function $y = f(2x - 6)$.

Question 12 — 1 mark — 112

Given $f(x) = \{(-3, 4), (2, 7), (8, 6)\}$, state the domain of the resulting function after $f(x)$ is reflected through the line $y = x$.

Question 13 — 3 marks — 113

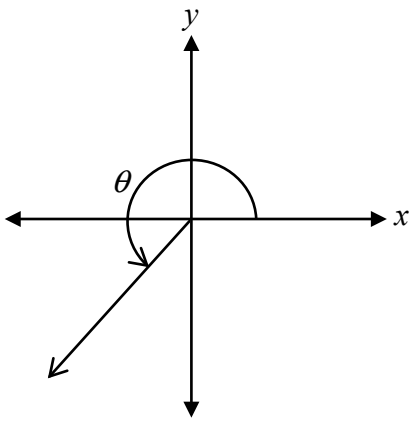
Determine the value of y in the following equation:

$$\log_x 27 - \log_x 3 = 2\log_x y$$

Question 14 — 1 mark — 114

Angle θ, measuring $\frac{5\pi}{4}$, is drawn in standard position as shown below.

Determine the measures of all angles in the interval $[-4\pi, 2\pi]$ that are coterminal with θ.



Question 15 — 3 marks — 115

Prove the identity below for all permissible values of x:

$$\frac{\sin^2 x}{\sec x + 1} = \cos x - \cos^2 x$$

Left-Hand Side	Right-Hand Side

Question 16 — 3 marks

116

Solve algebraically:

$${}_nC_2 = 4n + 5$$

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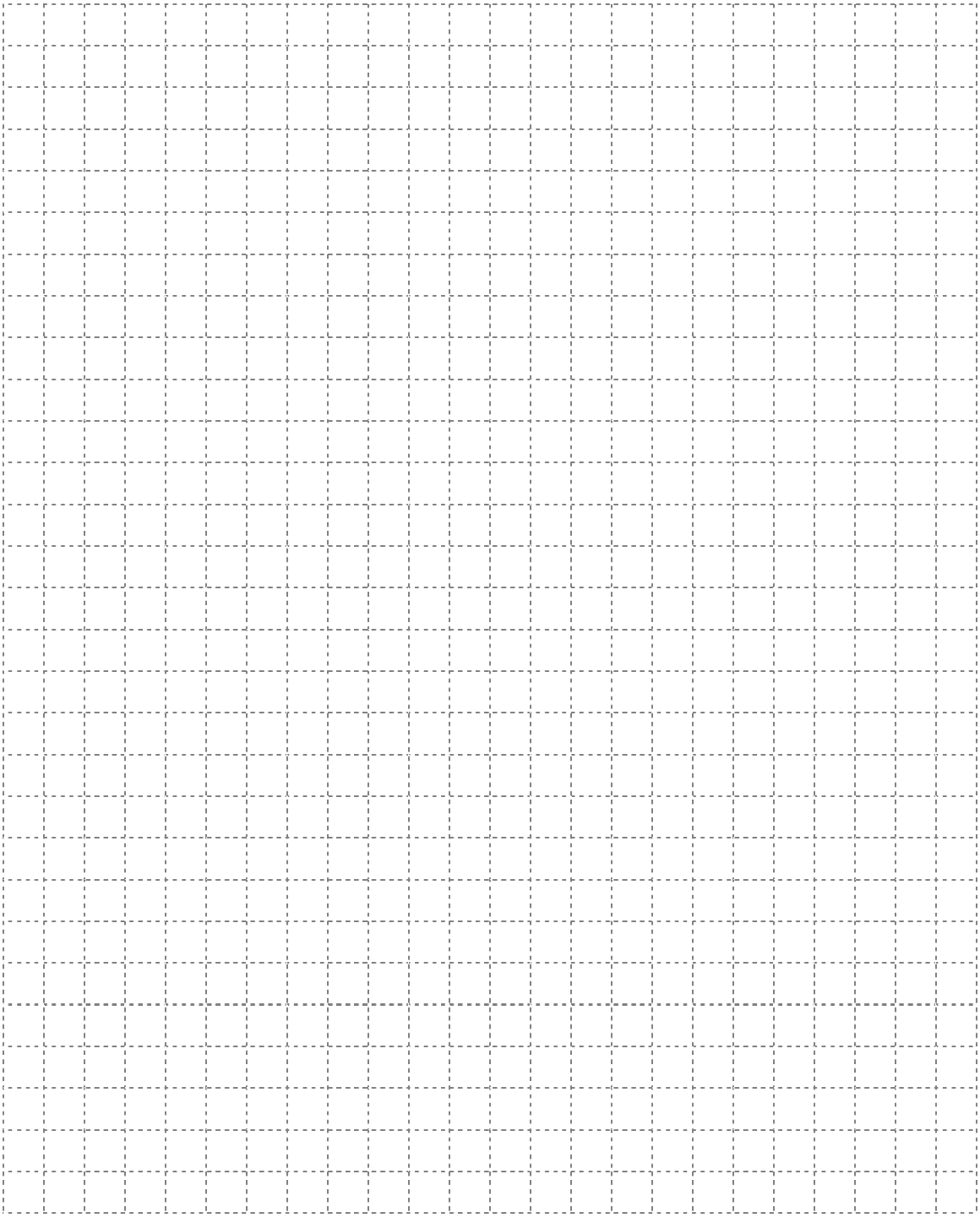

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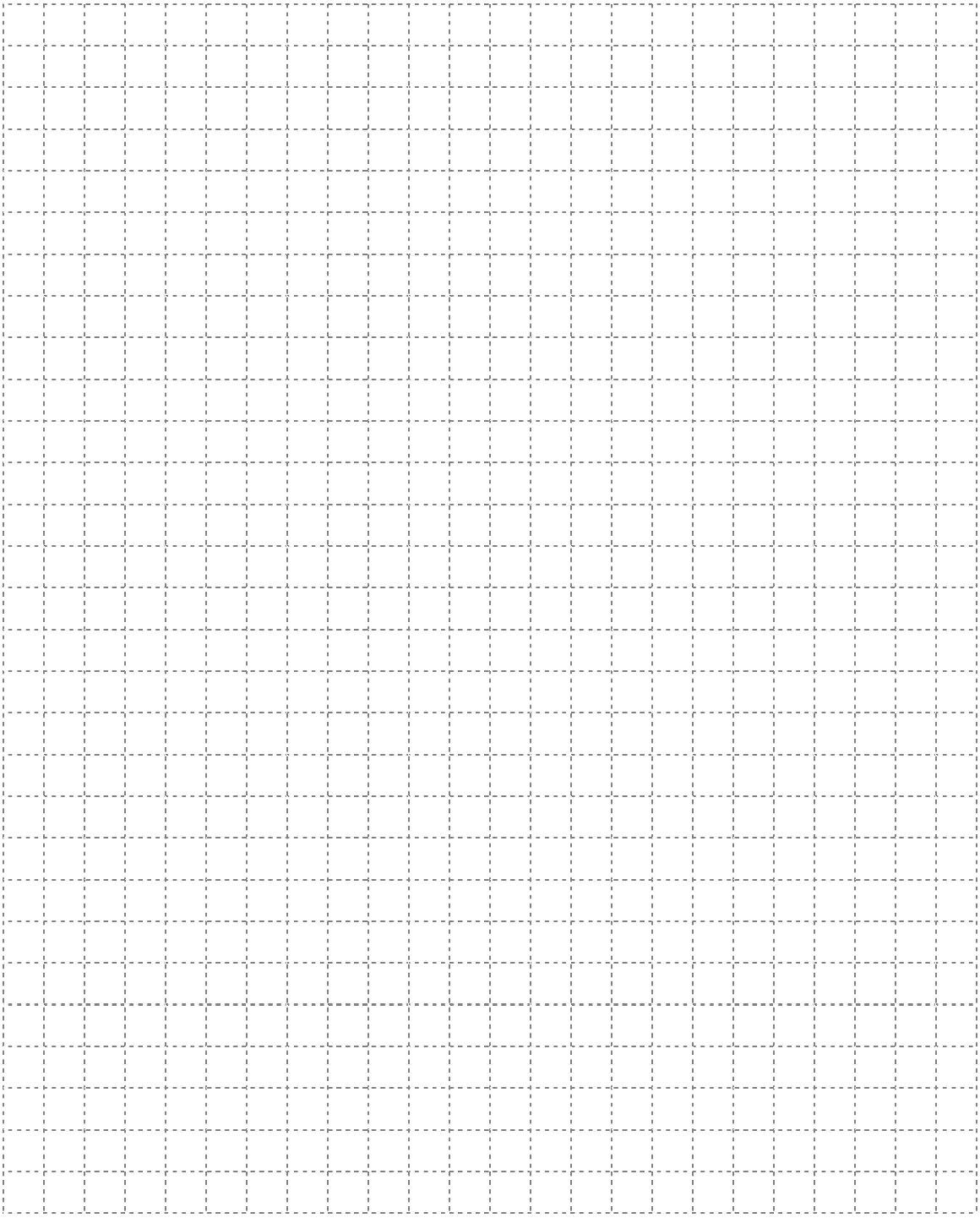

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