Grade 12
Pre-Calculus Mathematics
Achievement Test

# Booklet 2

June 2013

Manitoba

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# Instructions

## Multiple-Choice Questions

- There are 8 questions each worth one mark.
- Calculators are **not** allowed for this part of the test.
- You may use the spaces beside each question for rough work.
- Provide only one answer per question.
- There is no penalty for guessing.
- Record your answers on the sheet provided.

## Short and Long Answer Questions

- There are 21 questions worth a total of 44 marks.
- Calculators are **not** allowed for this part of the test.
- For full marks, your answer must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear, and well organized.
- Write each solution in the space provided.

No marks will be awarded for work done on this page.

## Question 17 — 1 mark

How many different arrangements are possible when arranging all of the letters of the word SEPTEMBER?

a) $9!$ b) $6!3!$ c) $\frac{9!}{3!}$ d) $\frac{6!}{3!}$

## Question 18 — 1 mark

Which one of the following angles terminates in Quadrant III?

a) 3 radians b) $\frac{7\pi}{5}$ radians c) $-210^\circ$ d) $500^\circ$

## Question 19 — 1 mark

There are 13 terms in the expansion of $(3x - y)^{2n}$. Determine the value of $n$.

a) 6 b) 6.5 c) 7 d) 26

## Question 20 — 1 mark

Which of the following is true about the periods of the three functions below?

$$f(\theta) = 2\sin 3\left(\theta - \frac{\pi}{2}\right) \qquad g(\theta) = \sin 3\theta + 6 \qquad k(\theta) = 3\sin\theta + 6$$

a) The graphs of $f(\theta)$ and $g(\theta)$ have the same period.

b) The graphs of $g(\theta)$ and $k(\theta)$ have the same period.

c) All of the graphs have the same period.

d) None of the graphs have the same period.

## Question 21 — 1 mark

Which of the following represents the general solution to the equation $\tan\theta = -1$?

a) $\theta = \frac{\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

b) $\theta = \frac{\pi}{4} + k\pi,\ k \in \mathrm{I}$

c) $\theta = \frac{3\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

d) $\theta = \frac{3\pi}{4} + k\pi,\ k \in \mathrm{I}$

## Question 22 — 1 mark

If $(3, -2)$ is a point on the graph of $y = f(x)$, what point must be on the graph of $y = 2f(x+1)$?

a) $(4, -1)$ b) $(4, -4)$ c) $(2, 1)$ d) $(2, -4)$

## Question 23 — 1 mark

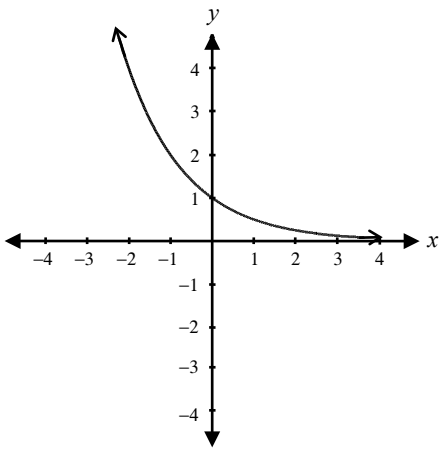
Which equation is represented by the graph sketched below?

a) $y = \left(\frac{1}{2}\right)^{-x}$

b) $y = \left(\frac{1}{2}\right)^{x}$

c) $y = 2^{x}$

d) $y = -2^{x}$




## Question 24 — 1 mark

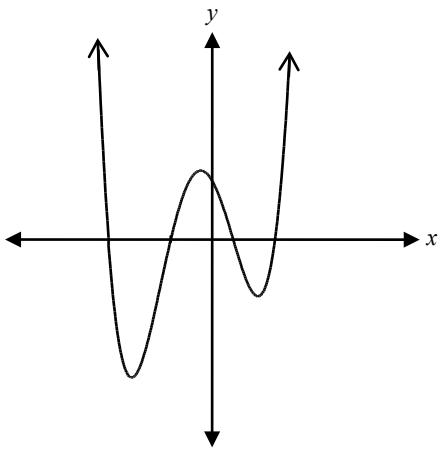
What is the degree of the polynomial represented below?

a) 2

b) 3

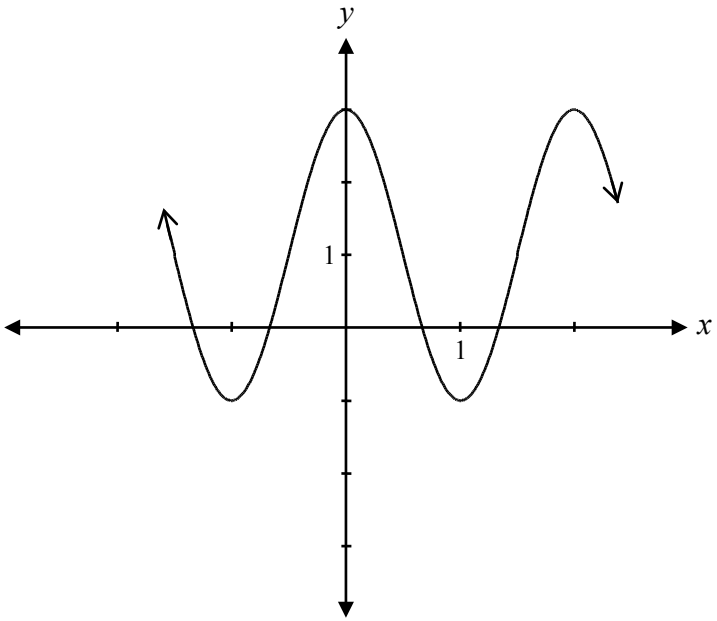
c) 4

d) 5



## Question 25 — 1 mark — 117

Given the graph of $y = 2\cos \pi x + 1$ below, determine another equation that will produce the same graph.




$y =$ _____

## Question 26 — 2 marks — 118

Given $f(x) = 3$ and $g(x) = x + 2,$ determine the domain and range of $h(x) = \dfrac{f(x)}{g(x)}$.

Domain: _____

Range: _____

## Question 27 — 2 marks — 119

Explain how to find the exact value of $\sec\left(\frac{19\pi}{6}\right)$.

## Question 28 — 1 mark — 120

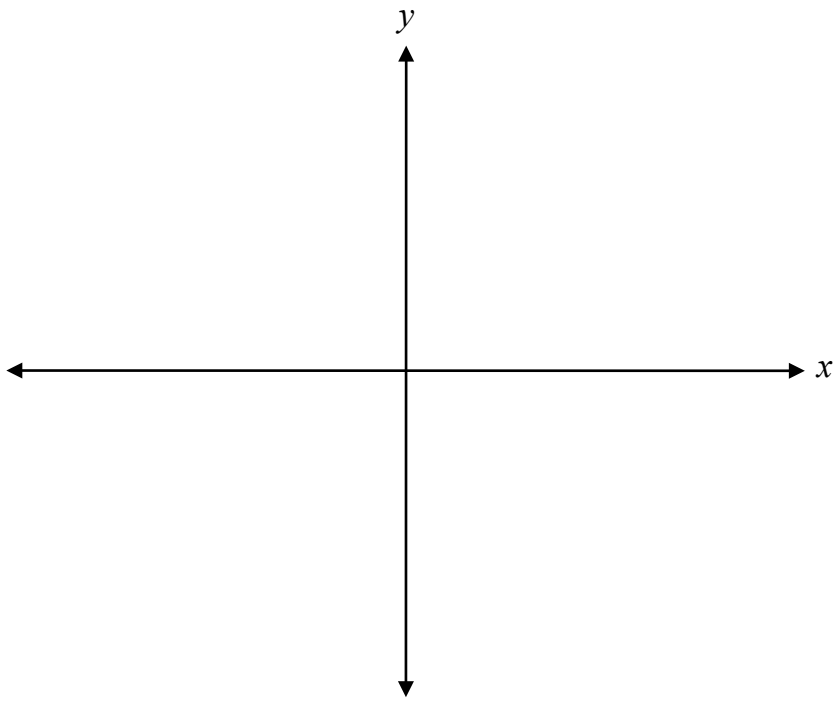
Given $f(x) = 4 - x,$ verify that $f^{-1}(x) = f(x).$

## Question 29 — 3 marks — 121

Sketch the graph of:

$$f(x) = (2 - x)(x + 3)(x + 1)^2$$

Label the $x$-intercepts and $y$-intercept.



## Question 30 — 1 mark — 122

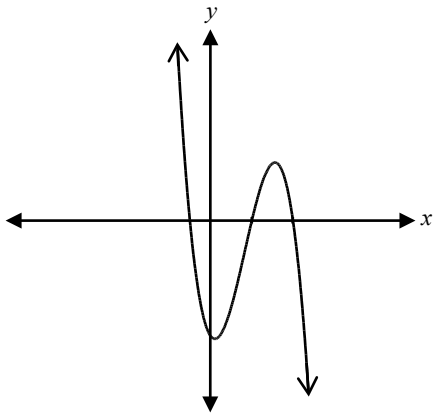
Which expression has a larger value?

$$\log_2 36 \quad \text{or} \quad \log_3 80$$

Justify your answer.

## Question 31 — 1 mark — 123

The graph below represents the equation $y = ax^3 + 6x^2 + 5x - 10$.




What must be true about the value of $a$? Explain your reasoning.

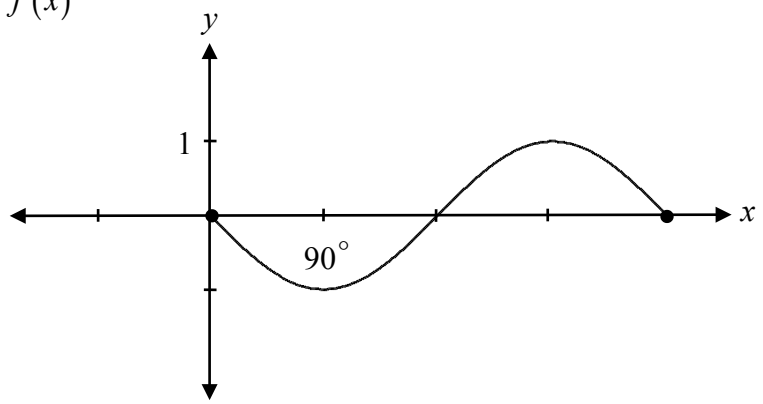
## Question 32 — 2 marks — 124

The terminal arm of an angle $\theta$, in standard position, intersects the unit circle in Quadrant IV at a point $\mathrm{P}\left(\frac{\sqrt{5}}{4}, y\right)$. Determine the value of $\sin\theta$.

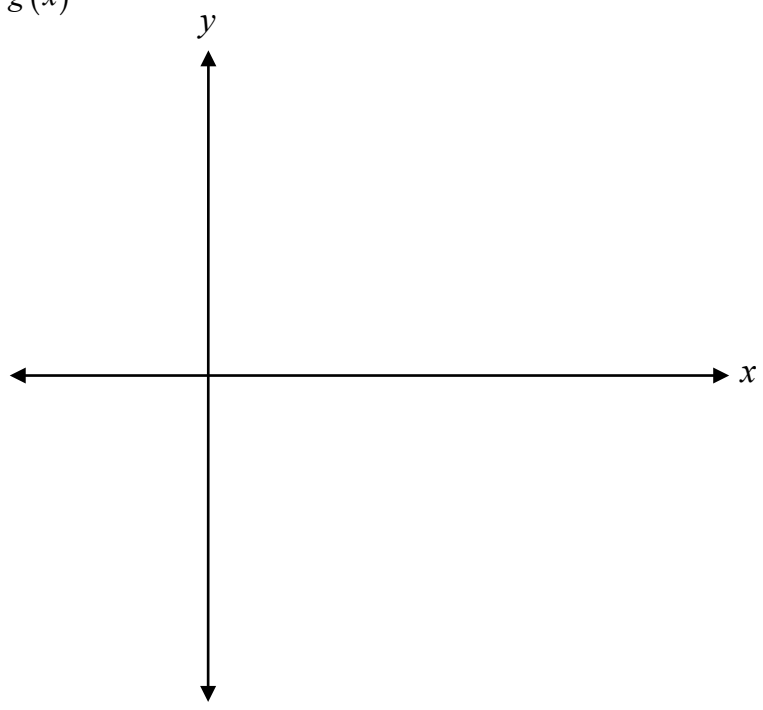
## Question 33 — 2 marks — 125

Given the sinusoidal function $f(x)$ below, sketch the graph of $g(x) = |f(x)| - 1$.

$f(x)$




$g(x)$



## Question 34 — 2 marks — 126

The graph of a rational function, $f(x)$, has a point of discontinuity when $x = 2$ and an asymptote when $x = 4$. Write a possible equation for $f(x)$.

## Question 35 — 2 marks — 127

Given that $(x - 1)$ is one of the factors, express $x^3 - 57x + 56$ as a product of factors.

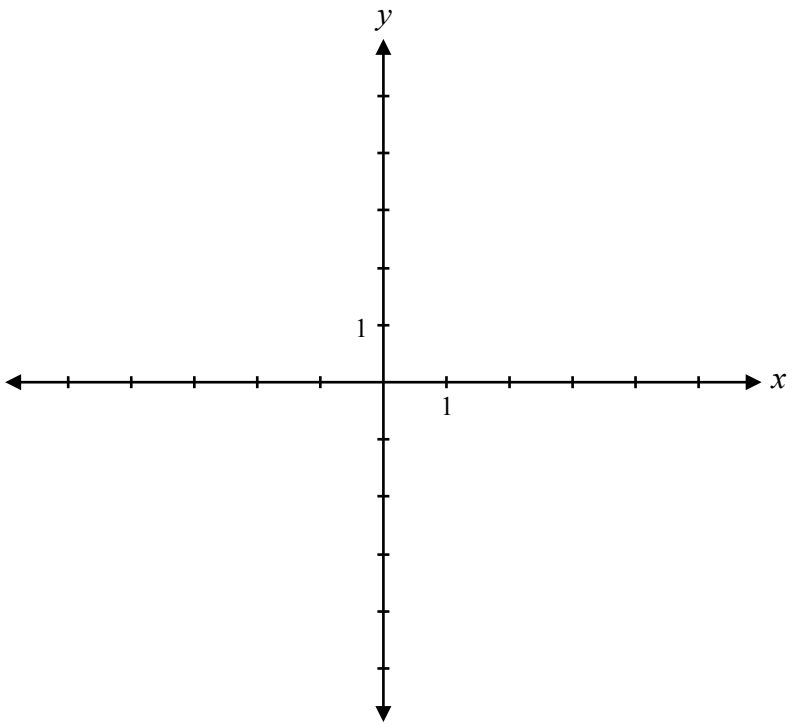
## Question 36 — 2 marks — 128

Give an example using values for $A$ and $B$, in degrees or radians, to verify that $\cos(A + B) = \cos A + \cos B$ is **not** an identity.

| Left-Hand Side | Right-Hand Side |
| --- | --- |
| | |

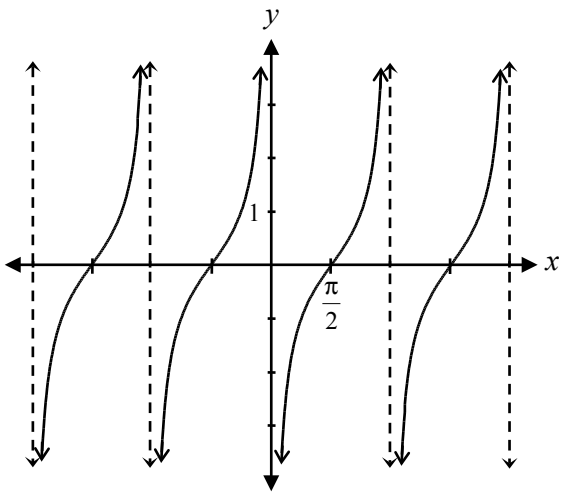
## Question 37 — 3 marks — 129

Sketch the graph of $y = \sqrt{x+1} - 2$ and verify that the value of the $x$-intercept is the same as the solution to the equation $\sqrt{x+1} - 2 = 0$.



## Question 38 — 1 mark — 130

Mohamed is asked to sketch the graph of $y = \tan x$.
His graph is shown below.




Explain why his graph is incorrect.

## Question 39 — 2 marks — 131

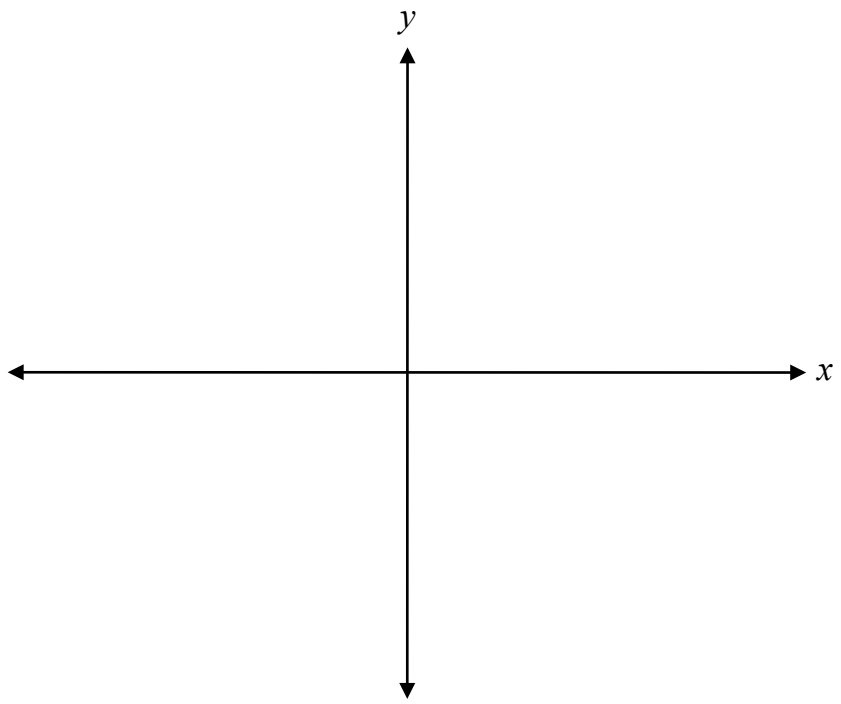
On the interval $0 \leq \theta < 2\pi,$ identify the non-permissible values of $\theta$ for the trigonometric identity:

$$\tan\theta = \frac{1}{\cot\theta}$$

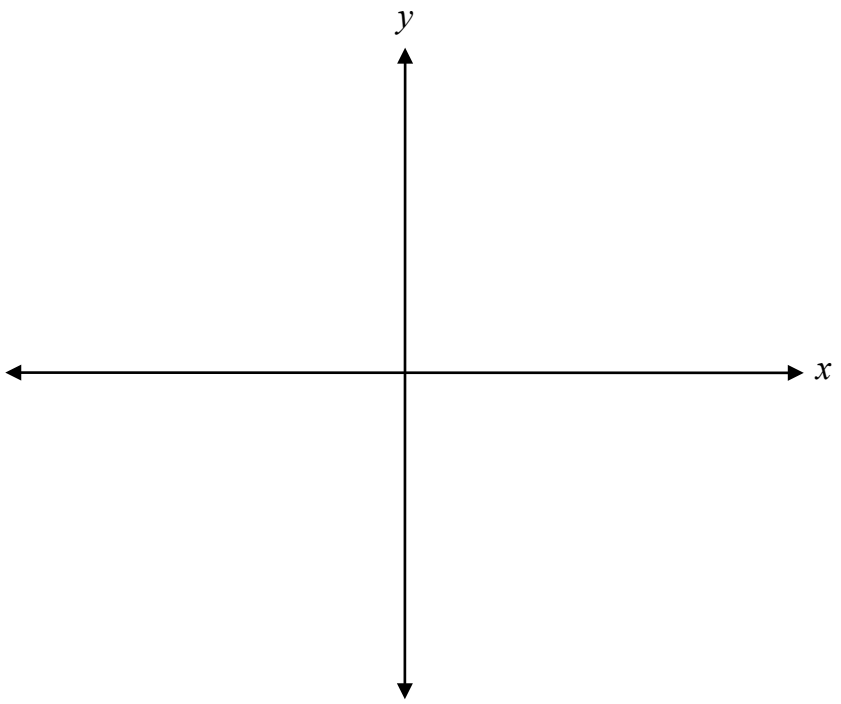
## Question 40 — a) 2 marks b) 2 marks

132
133

a) Sketch the graph of $y = \ln(x)$.




b) Sketch the graph of $y = -\ln(x - 2)$.



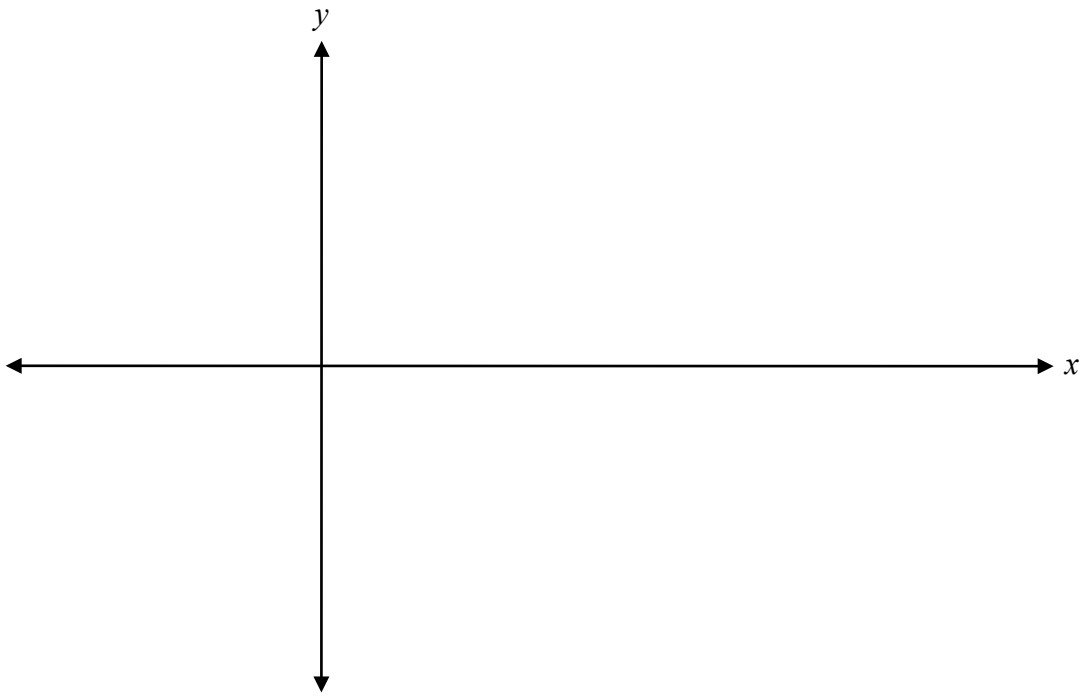
## Question 41 — 2 marks — 134

Given $f(x) = \sqrt{x-2}$ and $g(x) = 3x$, write the equation for $h(x) = f(g(x))$.

What are the restrictions on the domain of $h(x)$?
Explain your reasoning.

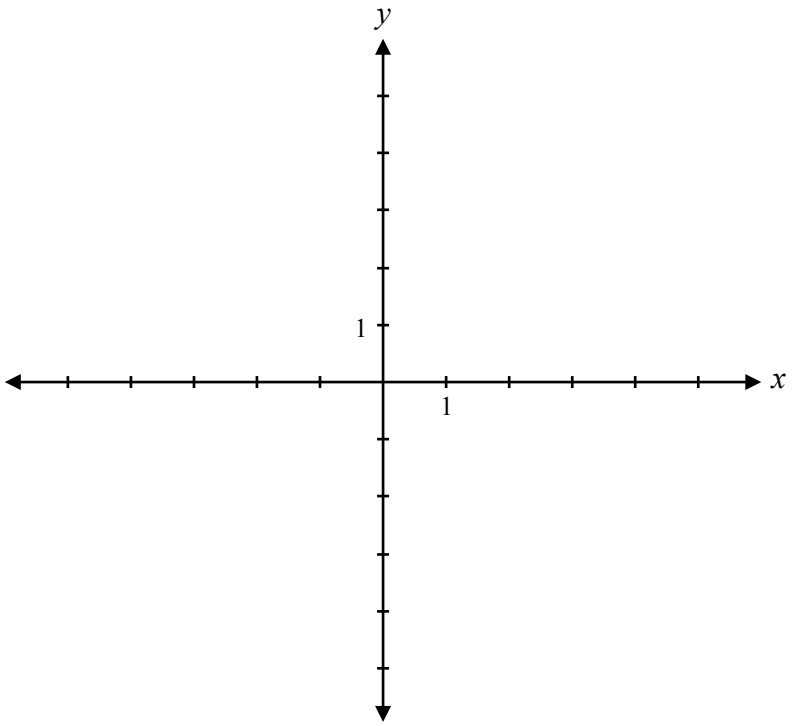
## Question 42 — 3 marks — 135

Sketch the graph of $y = 10\cos\left[\frac{\pi}{2}(x - 2)\right]$ over the interval $[0, 6]$.



## Question 43 — 4 marks — 136

Sketch the graph of the function $f(x) = \dfrac{x^2}{x^2 - x}$.



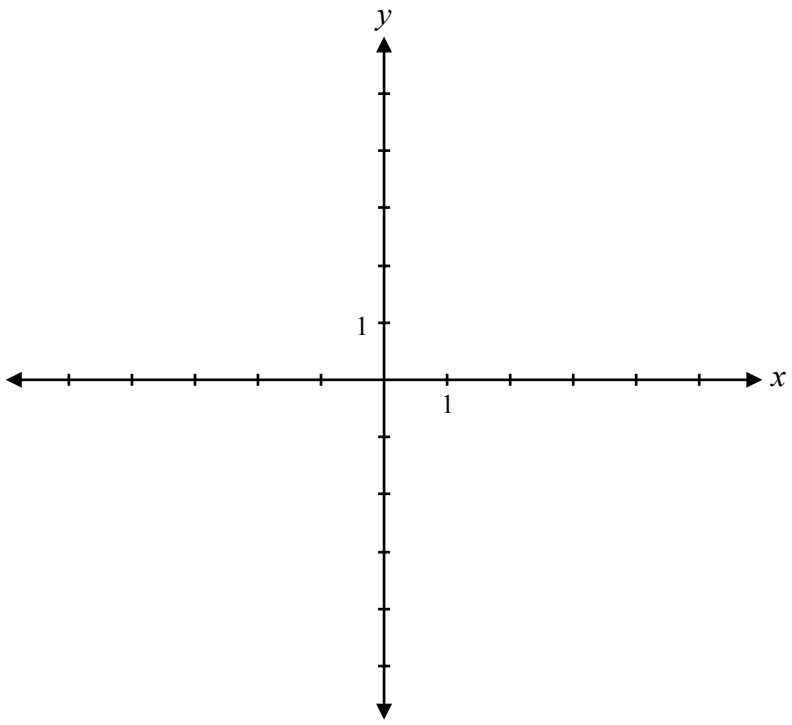
## Question 44 — 2 marks — 137

Is $(x-3)$ a factor of $x^4 - x^3 - 3x^2 + x - 1$?

Justify your answer.

## Question 45 — 2 marks — 138

Given $f(x) = x - 1$ and $g(x) = x^2$, write the equation of $y = f(g(x))$ and sketch the graph.



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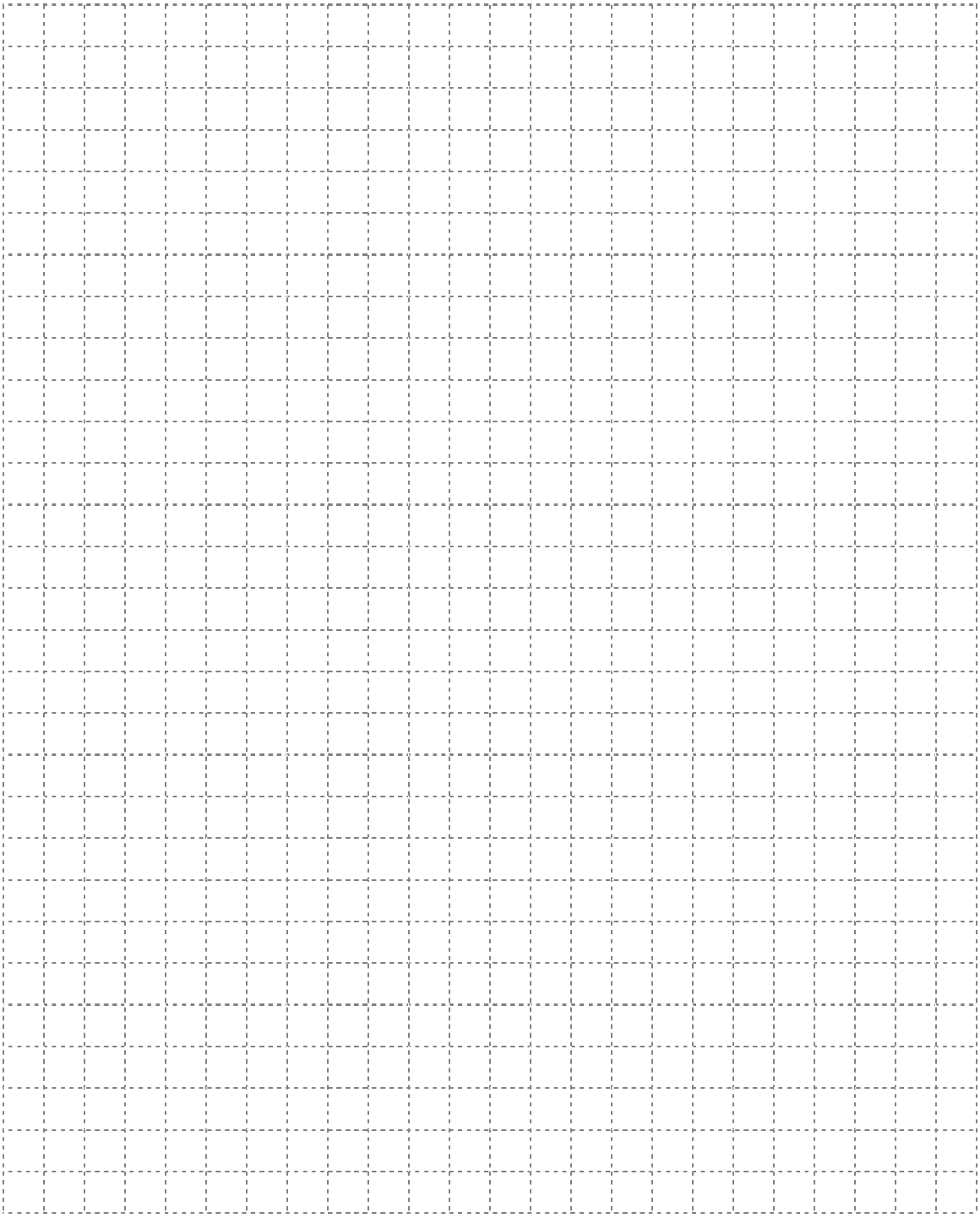

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