Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2014

Manitoba

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.

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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
Booklet 1*	10 Short-Answer	17	35
	5 Long-Answer	18	
Booklet 2	10 Multiple-Choice	10	57
	10 Short-Answer	15	
	9 Long-Answer	32	
Total			**92**

* The first 6 questions in *Booklet 1* require a calculator. 
You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*.
These words are explained below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

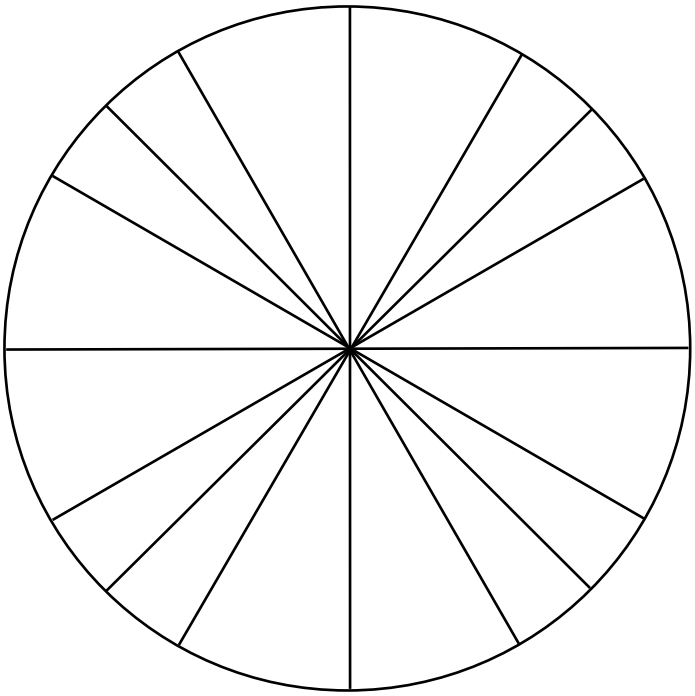
Solve: Give a solution for a problem or determine the value(s) of a variable.

Verify: Establish the truth of a statement by substitution or comparison.

Scrap Paper

No marks will be awarded for work done on this page.

Unit Circle (can be used if needed)



Scrap Paper

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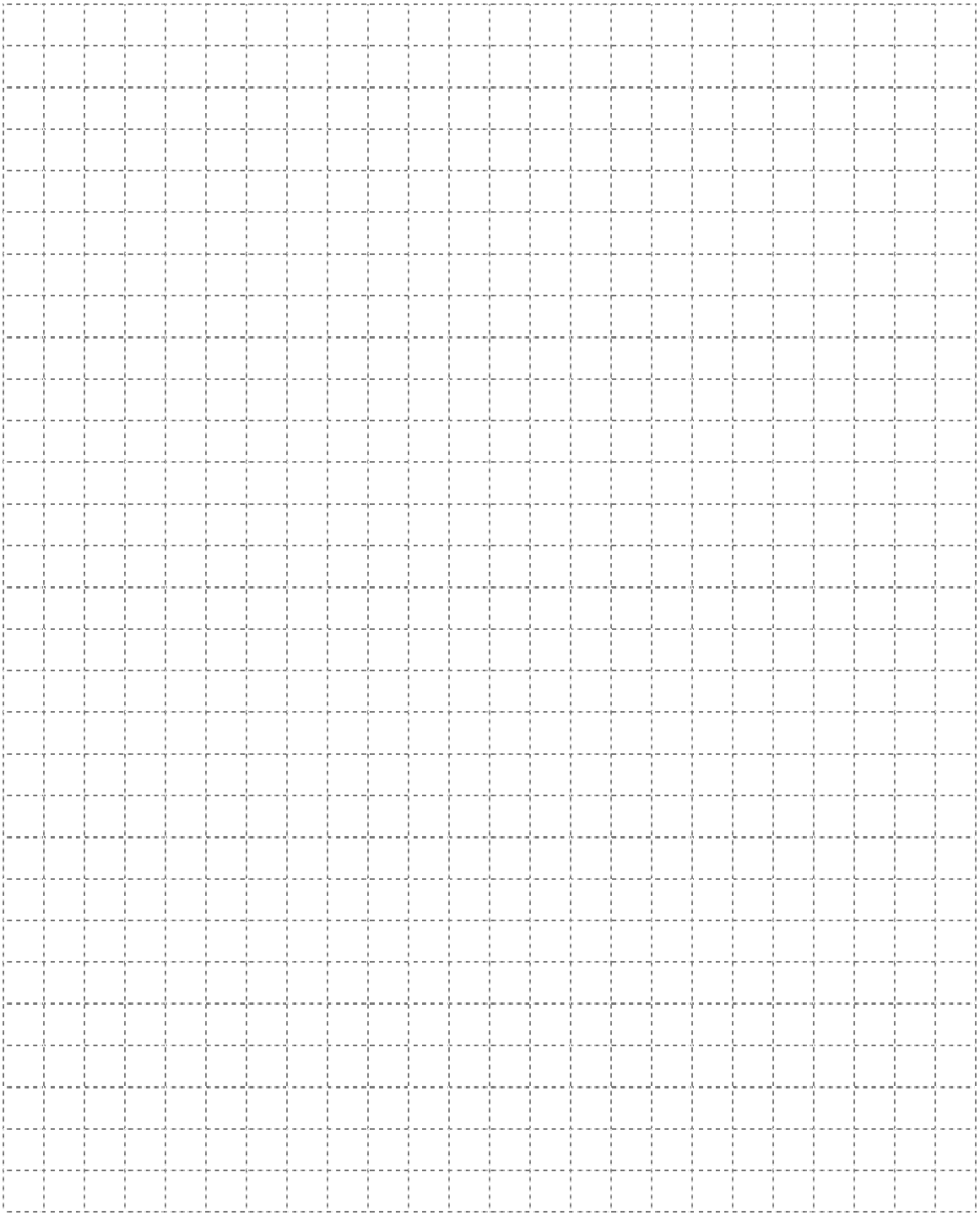


Instructions

- There are 15 questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

No marks will be awarded for work done on this page.

Question 1  2 marks 101

Find the coterminal angle to $\frac{27\pi}{5}$ over the interval $\left[-360^\circ, 0^\circ\right)$.

Question 2  3 marks

Solve the following equation over the interval $0 \le \theta < 2\pi$.

$$(\tan\theta - 3)(\tan\theta + 1) = 0$$

Question 3  2 marks 103

An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

You may use the formula below:

$$M = \log\left(\frac{A}{A_0}\right)$$

where M is the magnitude of the earthquake on the Richter scale

A is the intensity of the earthquake

A_0 is the intensity of a standard earthquake

Express your answer as a whole number.

Question 4  2 marks 104

Find and simplify the last term in the expansion of $(2y - 3x)^7$.

Question 5  3 marks 105

Given $\log_a 9 = 1.129$ and $\log_a 4 = 0.712,$ find the value of $\log_a 12.$

Question 6  2 marks 106

How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

Note: A calculator is not required for the remaining test questions.

Question 7 — 4 marks — 107

Solve the following equation over the interval $[0, 2\pi]$.

$$2\cos 2\theta - 1 = 0$$

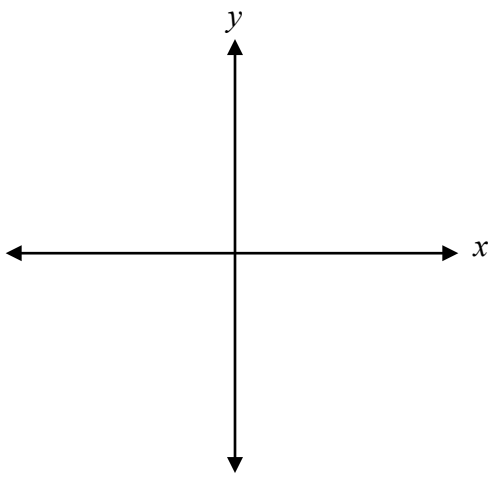
Question 8 — 1 mark — 108

Alex incorrectly explains to Rashid that the graph of $y = 2f(x) + 5$ means you first move the graph of $y = f(x)$ up 5 units and then multiply the y values by 2.

Explain to Rashid the correct way to transform the graph.

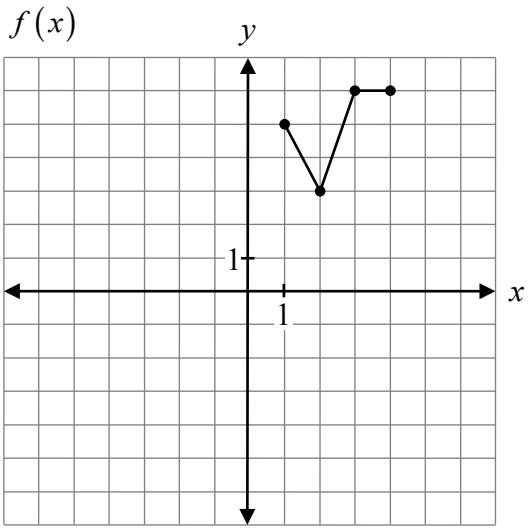
Question 9 — 1 mark — 109

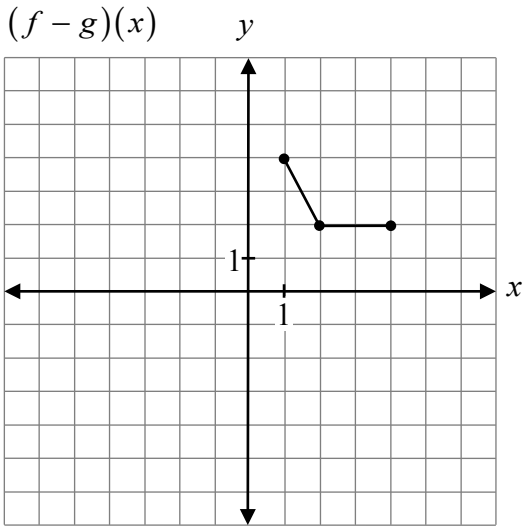
Sketch the angle of 5 radians in standard position.

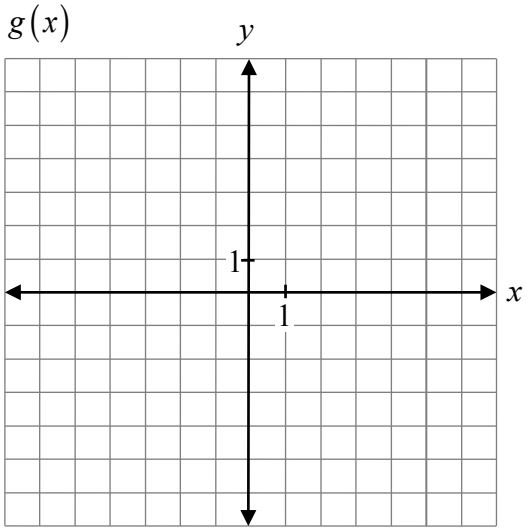


Question 10 — 2 marks — 110

Given the graphs of $f(x)$ and $(f-g)(x)$, sketch the graph of $g(x)$.







Question 11 — 2 marks

111

A particular math class has a large number of students. From this class, you are to create a committee of 4 students that has at least 1 girl.

Without actually solving the problem, explain the strategy you would use to find the total number of ways to select this committee.

Question 12 — a) 2 marks b) 2 marks

112
113

a) Prove the identity below for all permissible values of θ.

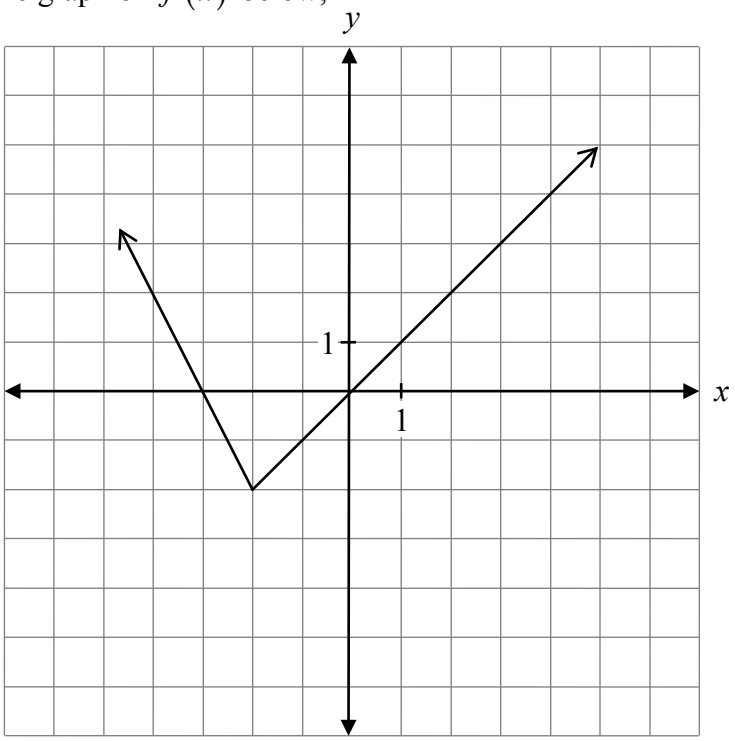
$$\frac{1+2\cos^2\theta}{\cos^2\theta} = \tan^2\theta + 3$$

Left-Hand Side	Right-Hand Side

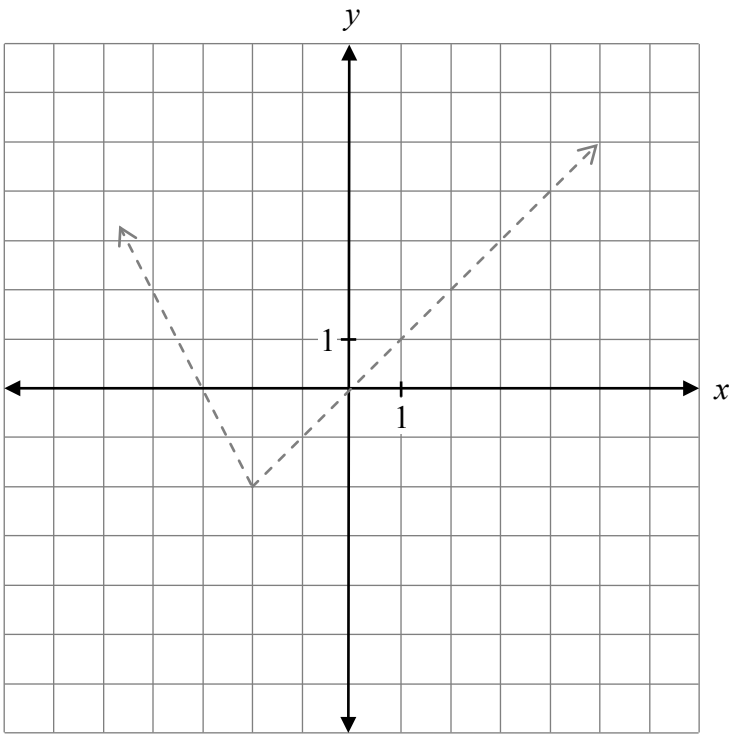
b) Determine all the non-permissible values for θ.

Question 13 — 2 marks — 114

Given the graph of $f(x)$ below,



sketch the graph of $g(x) = f(x-2)+3$.



The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 14 — a) 3 marks b) 1 mark

115
116

Given that $\sin\alpha = \frac{5}{13}$, where α is in Quadrant II, and $\cos\beta = \frac{2}{5}$, where β is in Quadrant IV, find the exact value of:

a) $\cos(\alpha + \beta)$

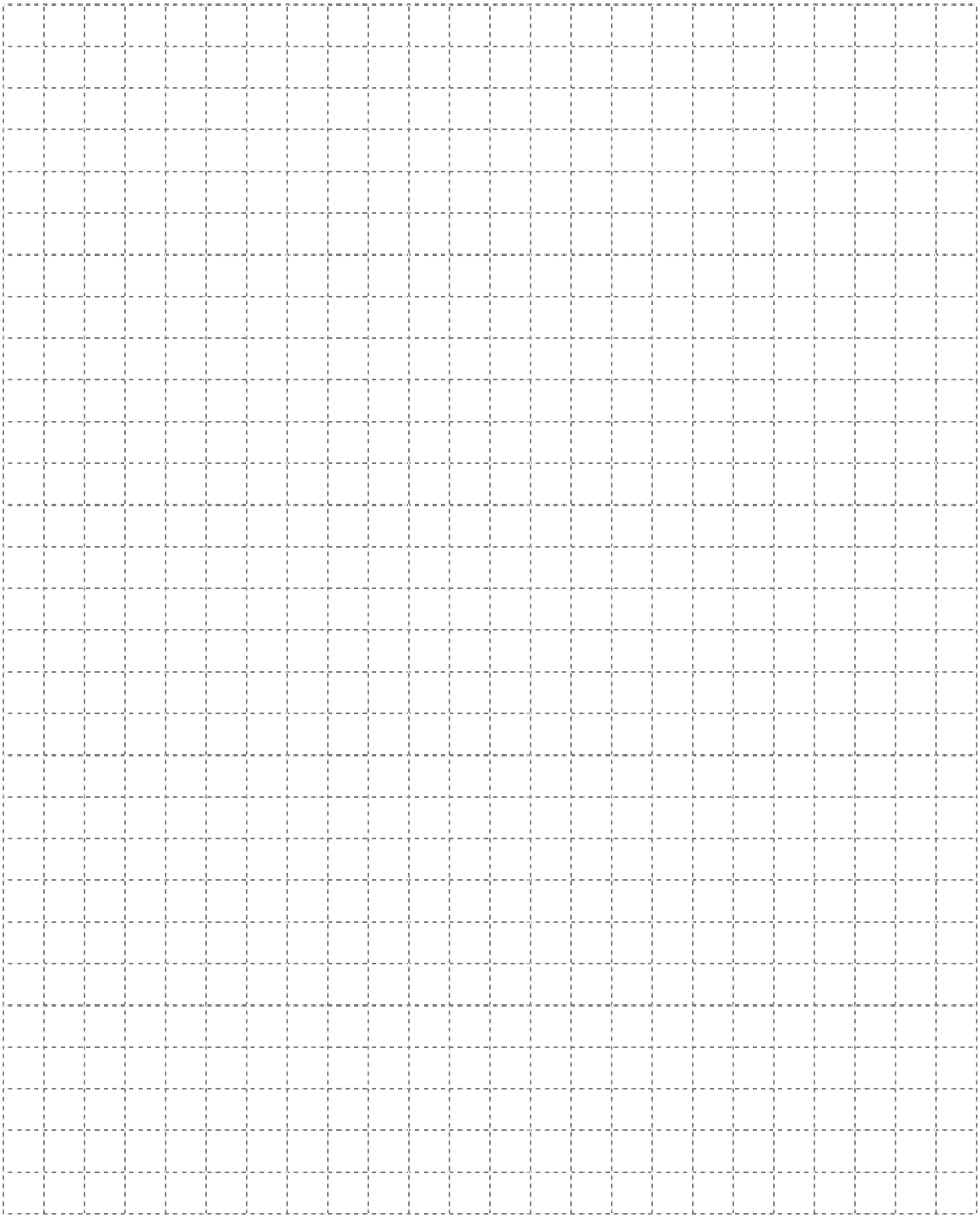
b) $\sin 2\alpha$

Question 15 — 1 mark

117

If $f(x) = x^3$ and $g(x) = 2x - 3,$ what is the value of $\left(\frac{f}{g}\right)(-1)$?

No marks will be awarded for work done on this page.



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