Lesson 6.5 Exercises, pages 521–526

A

3. Identify the indicated characteristic of each function.

a) amplitude of $y = 5 \sin x$

The amplitude is 5.

b) amplitude of $y = 2 \cos x$

The amplitude is 2.

c) period of $y = \sin 10x$

The period is: $\frac{2\pi}{10} = \frac{\pi}{5}$

d) period of $y = \tan 4x$

The period is: $\frac{\pi}{4}$

e) phase shift of $y = \sin\left(x - \frac{\pi}{7}\right)$

The phase shift is: $\frac{\pi}{7}$

f) phase shift of $y = \cos\left(x + \frac{\pi}{12}\right)$

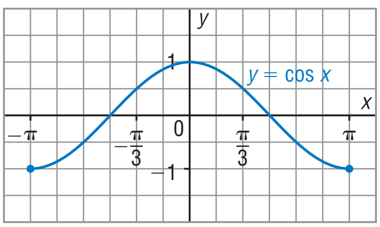
The phase shift is: $-\frac{\pi}{12}$

B

4. For each function below, sketch the graph for $-\pi \le x \le \pi$, then identify each characteristic:

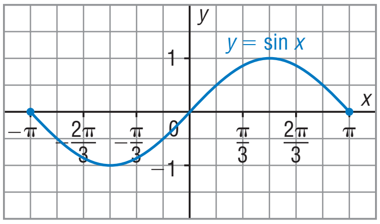
i) amplitude
ii) period
iii) zeros
iv) equations of any asymptotes
v) domain of the function
vi) range of the function

a) $y = \cos x$



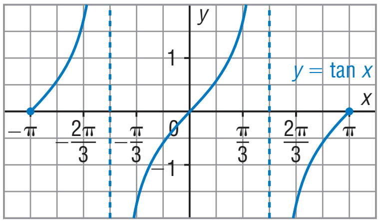
i) The amplitude is 1.
ii) The period is 2π.
iii) The zeros are $\pm\frac{\pi}{2}$.
iv) There are no asymptotes.
v) The domain is $x \in \mathbb{R}$.
vi) The range is $-1 \le y \le 1$.

b) $y = \sin x$



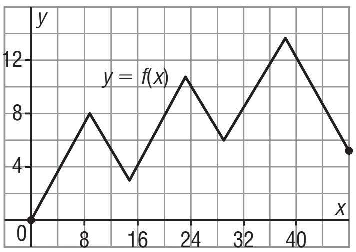
i) The amplitude is 1.
ii) The period is 2π.
iii) The zeros are $0, \pm\pi$.
iv) There are no asymptotes.
v) The domain is $x \in \mathbb{R}$.
vi) The range is $-1 \leq y \leq 1$.

c) $y = \tan x$



i) There is no amplitude.
ii) The period is π.
iii) The zeros are $0, \pm\pi$.
iv) The asymptotes are $x = \pm\frac{\pi}{2}$.
v) The domain is $x \neq \pm\frac{\pi}{2}$.
vi) The range is $y \in \mathbb{R}$.

5. Does this graph represent a periodic function? Explain.



No, the graph does not represent a periodic function because the graph does not repeat in regular intervals.

6. Use technology.

a) **i)** Graph each function.

$y = 2 \cos x \qquad y = -3 \cos x \qquad y = \frac{1}{3} \cos x$

ii) How does varying the value of a affect the graph of $y = a \cos x$?

When $a = 1$, the graph is $y = \cos x$ with amplitude 1. As a varies, the amplitude varies. When $a > 1$, the graph of $y = \cos x$ is stretched vertically by a factor of a and the amplitude increases; when $0 < a < 1$, the graph of $y = \cos x$ is compressed vertically by a factor of a and the amplitude decreases; when $a < 0$, the graph is also reflected in the x-axis.

b) **i)** Graph each function.

$y = \sin 3x \qquad y = \sin(-4x) \qquad y = \sin \frac{3}{4}x$

ii) How does varying the value of b affect the graph of $y = \sin bx$?

When $b = 1$, the graph is $y = \sin x$ and its period is 2π. As b varies, the period of the graph varies. When $b > 1$, the graph of $y = \sin x$ is compressed horizontally by a factor of $\frac{1}{b}$ and the period decreases; when $0 < b < 1$, the graph of $y = \sin x$ is stretched horizontally by a factor of $\frac{1}{b}$ and the period increases; when $b < 0$, the graph is also reflected in the y-axis.

c) **i)** Graph each function.

$y = \cos\left(x - \frac{\pi}{6}\right) \qquad y = \cos\left(x - \frac{\pi}{4}\right) \qquad y = \cos\left(x + \frac{\pi}{3}\right)$

ii) How does varying the value of c affect the graph of $y = \cos(x - c)$?

When $c = 1$, the graph is $y = \cos x$ with phase shift 0. As c varies, the phase shift varies. When $c > 0$, the graph of $y = \cos x$ is translated c units right; when $c < 0$, the graph is translated c units left.

d) **i)** Graph each function.

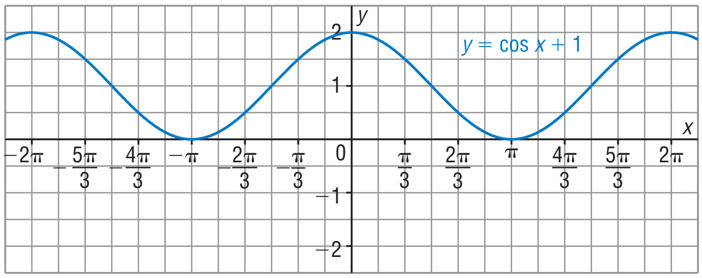
$y = \sin x + 1 \qquad y = \sin x - 2 \qquad y = \sin x + 0.5$

ii) How does varying the value of d affect the graph of $y = \sin x + d$?

When $d = 0$, the graph is $y = \sin x$. As d varies, the graph of $y = \sin x$ is translated vertically. When $d > 0$, the graph is translated d units up; when $d < 0$, the graph is translated d units down.

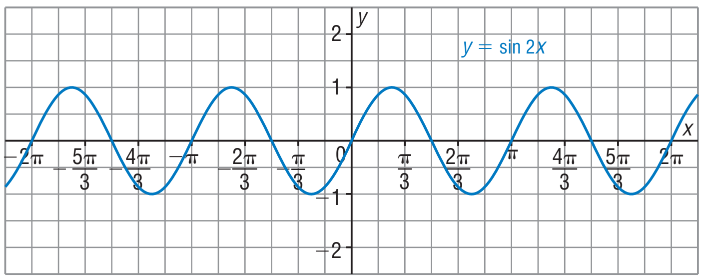
7. Sketch the graph of each function. Describe your strategy.

a) $y = \cos x + 1$



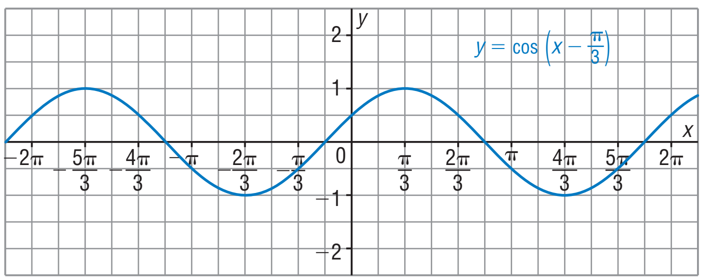
I used the completed table of values for $y = \cos x$ from Lesson 6.4, translated each point 1 unit up, extended the pattern, then drew a smooth curve through the points.

b) $y = \sin 2x$



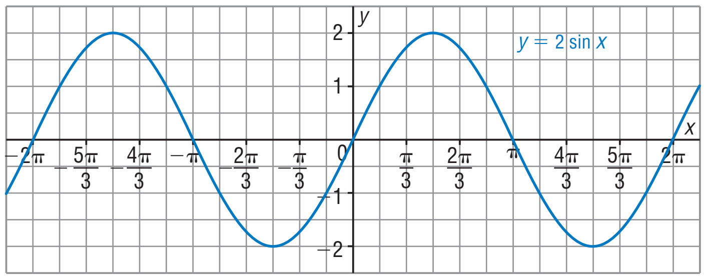
I used the completed table of values for $y = \sin x$ from Lesson 6.4, halved each x-coordinate, extended the pattern, then drew a smooth curve through the points.

c) $y = \cos\left(x - \frac{\pi}{3}\right)$



I used the completed table of values for $y = \cos x$ from Lesson 6.4, translated each point $\frac{\pi}{3}$ units right, extended the pattern, then drew a smooth curve through the points.

d) $y = 2 \sin x$

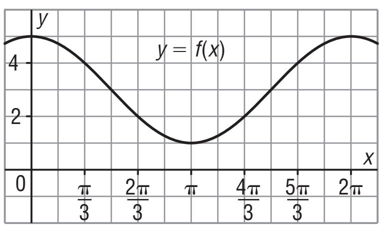


I used the completed table of values for $y = \sin x$ from Lesson 6.4, doubled each y-coordinate, extended the pattern, then drew a smooth curve through the points.

8. Use technology to graph $y = \sin\left(x + \frac{\pi}{2}\right)$ and $y = \cos x$. Explain the result.

The graphs coincide. The graph of $y = \cos x$ is the image of the graph of $y = \sin x$ after a horizontal translation of $\frac{\pi}{2}$ units left; that is, for any angle x radians, $\cos x = \sin\left(x + \frac{\pi}{2}\right)$.

9. A student says that the amplitude of this sinusoidal function is 5. Is the student correct? Explain.

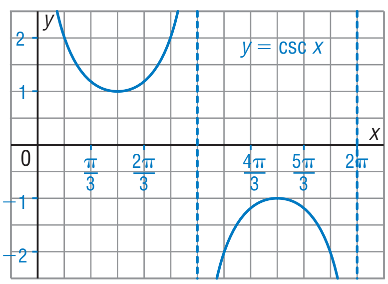


No, the amplitude is one-half of the vertical distance between a maximum point and a minimum point, which is 2.

C

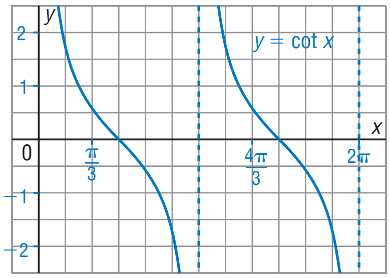
10. Sketch the graph of each function. Identify its characteristics.

a) $y = \csc x$



Take the reciprocal of each x-value in the completed table for $y = \sin x$ in Lesson 6.4, plot the points, extend the pattern, then join the points with 2 smooth curves. There is no amplitude. The period is 2π. There are no zeros. The equations of the asymptotes are $x = k\pi, k \in \mathbb{Z}$. The domain is $x \neq k\pi, k \in \mathbb{Z}$. The range is $y \geq 1$ or $y \leq -1$.

b) $y = \cot x$



Take the reciprocal of each x-value in the completed table for $y = \tan x$ in Lesson 6.4, plot the points, extend the pattern, then join the points with 2 smooth curves. There is no amplitude. The period is π. The zeros are $(2k + 1)\frac{\pi}{2}$, $k \in \mathbb{Z}$. The equations of the asymptotes are $x = k\pi$, $k \in \mathbb{Z}$. The domain is $x \neq k\pi$, $k \in \mathbb{Z}$. The range is $y \in \mathbb{R}$.

11. Use technology. Graph the function $y = \sin x + \cos x$. Is it periodic? Explain. Is it sinusoidal? Explain.

The function is periodic because its values repeat at regular intervals. The function is sinusoidal because its maximum and minimum values are equidistant from the centre line.