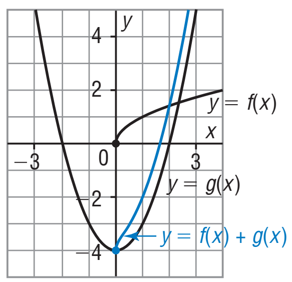
REVIEW, pages 327–334

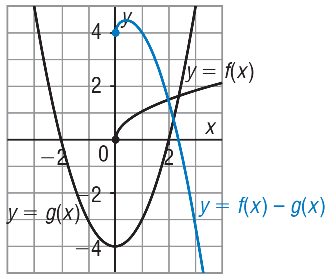
4.1

1. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each function below, then identify its domain and range. Estimate the range if necessary.

a) $y = f(x) + g(x)$



b) $y = f(x) - g(x)$



From the graphs:

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$
−3	–	5	–	–
−2	–	0	–	–
−1	–	−3	–	–
0	0	−4	−4	4
1	1	−3	−2	4
2	$\doteq 1.4$	0	$\doteq 1.4$	$\doteq 1.4$
3	$\doteq 1.7$	5	$\doteq 6.7$	$\doteq -3.3$

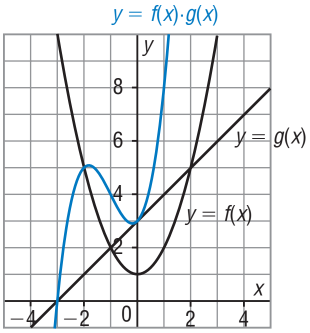
Plot points at: (0, −4), (1, −2), (2, 1.4)
Join the points with a smooth curve.
Domain: $x \geq 0$
Range: $y \geq 4$

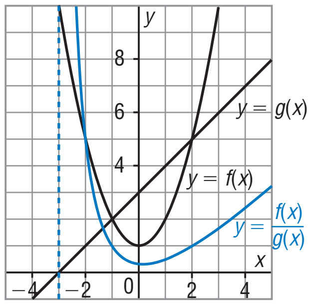
Plot points at: (0, 4), (1, 4), (2, 1.4), (3, −3.3)
Join the points with a smooth curve.
Domain: $x \geq 0$
From the graph, the approximate range is: $y \leq 4.5$

2. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each function below.

a) $y = f(x) \cdot g(x)$

b) $y = \dfrac{f(x)}{g(x)}$





From the graphs:

x	$f(x)$	$g(x)$	$f(x) \cdot g(x)$	$\dfrac{f(x)}{g(x)}$
−3	10	0	0	undefined
−2	5	1	5	5
−1	2	2	4	1
0	1	3	3	$0.\overline{3}$
1	2	4	8	0.5
2	5	5	25	1
3	10	6	60	$1.\overline{6}$

Plot points at: (−3, 0), (−2, 5), (−1, 4), (0, 3), (1, 8)
Join the points with a smooth curve.

Plot points at: (−2, 5), (−1, 1), $(0, 0.\overline{3})$, (1, 0.5), (2, 1), $(3, 1.\overline{6})$
Draw a vertical asymptote through $x = -3$.
Join the points with a smooth curve.

4.2

3. Given $f(x) = 3x - 4$ and $g(x) = |2 - 4x|$, write an explicit equation for each function below then determine its domain and range.

a) $h(x) = f(x) + g(x)$

$h(x) = 3x - 4 + |2 - 4x|$
The domain is: $x \in \mathbb{R}$
Use graphing technology.
From the graph of the function, the range is: $y \geq -2.5$

b) $d(x) = f(x) - g(x)$

$d(x) = 3x - 4 - |2 - 4x|$
The domain is: $x \in \mathbb{R}$
Use graphing technology.
From the graph of the function, the range is: $y \leq -2.5$

c) $p(x) = f(x) \cdot g(x)$

$p(x) = (3x - 4) \cdot |2 - 4x|$
The domain is: $x \in \mathbb{R}$
Use graphing technology.
From the graph of the function, the range is: $y \in \mathbb{R}$

d) $q(x) = \dfrac{f(x)}{g(x)}$

$q(x) = \dfrac{3x - 4}{|2 - 4x|}$
For the domain:
$2 - 4x \neq 0$
$x \neq 0.5$
The domain is: $x \neq 0.5$
Use graphing technology.
From the graph and table, the range is: $y < 0.75$

4. Given the function $h(x) = 3x^2 - 7x + 4$, write explicit equations for:

a) two functions $f(x)$ and $g(x)$ so that $h(x) = f(x) \cdot g(x)$

Sample response:
Factor: $h(x) = 3x^2 - 7x + 4$
$h(x) = (3x - 4)(x - 1)$
So, $f(x) = 3x - 4$ and $g(x) = x - 1$

b) three functions $f(x)$, $g(x)$, and $k(x)$ so that $h(x) = f(x) + g(x) + k(x)$

Sample response:
Write the given function as: $h(x) = 3x^2 + (-7x) + 4$
So, $f(x) = 3x^2$, $g(x) = -7x$, and $k(x) = 4$

c) three functions $f(x)$, $g(x)$, and $k(x)$ so that $h(x) = f(x) - g(x) - k(x)$

Sample response:
Write the given function as: $h(x) = 3x^2 - (7x) - (-4)$
So, $f(x) = 3x^2$, $g(x) = 7x$, and $k(x) = -4$

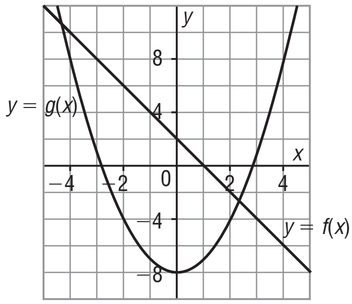
d) two functions $f(x)$ and $g(x)$ so that $h(x) = \dfrac{f(x)}{g(x)}$

Sample response:
Multiply and divide $h(x)$ by an expression that is never 0, such as $x^2 + 3$.

$h(x) = \dfrac{(x^2 + 3)(3x^2 - 7x + 4)}{x^2 + 3}$

So, $f(x) = (x^2 + 3)(3x^2 - 7x + 4)$ and $g(x) = x^2 + 3$

5. Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below.



a) $f(g(2))$

From the graph of $y = g(x)$, $g(2) = -4$
From the graph of $y = f(x)$, $f(-4) = 10$
So, $f(g(2)) = 10$

b) $g(f(2))$

From the graph of $y = f(x)$, $f(2) = -2$
From the graph of $y = g(x)$, $g(-2) = -4$
So, $g(f(2)) = -4$

c) $f(f(1))$

From the graph of $y = f(x)$, $f(1) = 0$
From the graph of $y = f(x)$, $f(0) = 2$
So, $f(f(1)) = 2$

d) $g(g(-2))$

From the graph of $y = g(x)$, $g(-2) = -4$
From the graph of $y = g(x)$, $g(-4) = 8$
So, $g(g(-2)) = 8$

6. Given the functions $f(x) = -2x + 3$, $g(x) = x^2 - 3x$, and $h(x) = \sqrt{x - 4}$, determine each value.

a) $f(g(1))$

$g(1) = 1^2 - 3(1)$
$= -2$
$f(g(1)) = f(-2)$
$= -2(-2) + 3$
$= 7$

b) $h(f(-3))$

$f(-3) = -2(-3) + 3$
$= 9$
$h(f(-3)) = h(9)$
$= \sqrt{9 - 4}$
$= \sqrt{5}$

c) $f(g(h(9)))$

$h(9) = \sqrt{9 - 4}$
$= \sqrt{5}$
$g(\sqrt{5}) = (\sqrt{5})^2 - 3\sqrt{5}$
$= 5 - 3\sqrt{5}$
$f(g(h(9))) = f(5 - 3\sqrt{5})$
$= -2(5 - 3\sqrt{5}) + 3$
$= -7 + 6\sqrt{5}$

d) $h(g(f(-0.5)))$

$f(-0.5) = -2(-0.5) + 3$
$= 4$
$g(4) = 4^2 - 3(4)$
$= 4$
$h(g(f(-0.5))) = h(4)$
$= 0$

7. Given $f(x) = 2x^2 - x$ and $g(x) = x + 4$, determine an explicit equation for each composite function then state its domain and range.

a) $f(g(x))$

$f(g(x)) = f(x + 4)$
$f(g(x)) = 2(x + 4)^2 - (x + 4)$
$f(g(x)) = 2x^2 + 16x + 32 - x - 4$
$f(g(x)) = 2x^2 + 15x + 28$
This is a quadratic function; its domain is: $x \in \mathbb{R}$
Use graphing technology to graph the function; its range is: $y \geq -0.125$

b) $g(f(x))$

$g(f(x)) = g(2x^2 - x)$
$g(f(x)) = 2x^2 - x + 4$
This is a quadratic function; its domain is: $x \in \mathbb{R}$
Use graphing technology to graph the function; its range is: $y \geq 3.875$

c) $g(g(x))$

$g(g(x)) = g(x + 4)$
$g(g(x)) = x + 4 + 4$
$g(g(x)) = x + 8$
This is a linear function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

d) $f(f(x))$

$f(f(x)) = f(2x^2 - x)$
$f(f(x)) = 2(2x^2 - x)^2 - (2x^2 - x)$
$f(f(x)) = 8x^4 - 8x^3 + 2x^2 - 2x^2 + x$
$f(f(x)) = 8x^4 - 8x^3 + x$
This is a quartic function; its domain is: $x \in \mathbb{R}$
Use graphing technology to graph the function; its range is: $y \geq -0.125$

8. Use composition of functions to determine whether $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 6$ are inverse functions.

Determine: $f(g(x)) = \frac{1}{2}(2x + 6) - 3$
$= x$

Determine: $g(f(x)) = 2\left(\frac{1}{2}x - 3\right) + 6$
$= x$

Since $f(g(x)) = g(f(x)) = x$, the functions are inverses.

4.4

9. Determine possible functions f and g so that $f(g(x)) = 2x^2 + 10x - 6$. Do this in two ways.

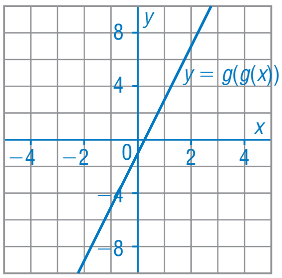
Sample response: Complete the square:
$f(g(x)) = 2x^2 + 10x - 6$
$f(g(x)) = 2(x^2 + 5x) - 6$
$f(g(x)) = 2(x^2 + 5x + 6.25 - 6.25) - 6$
$f(g(x)) = 2(x + 2.5)^2 - 18.5$

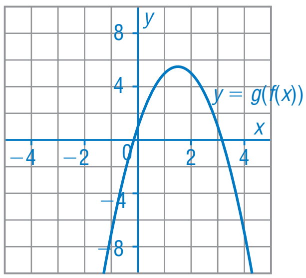
One way:
Replace $x + 2.5$ with x.
Let $g(x) = x + 2.5$, then
$f(x) = 2x^2 - 18.5$

A second way:
Replace $(x + 2.5)^2$ with x.
Let $g(x) = (x + 2.5)^2$, then
$f(x) = 2x - 18.5$

10. Given the functions $f(x) = x^2 - 3x$ and $g(x) = -2x + 1$, sketch a graph of each composite function below. Why are there no restrictions on x?

a) $y = g(g(x))$ **b)** $y = g(f(x))$





Make a table of values for the functions.

x	$f(x)$	$g(x)$	$g(g(x))$	$g(f(x))$
−1	4	3	−5	−7
0	0	1	−1	1
1	−2	−1	3	5
2	−2	−3	7	5
3	0	−5	11	1
4	4	−7	15	−7

a) Graph the points with coordinates $(x, g(g(x)))$ that fit on the grid. Draw a line through the points for the graph of $y = g(g(x))$.
b) Graph the points with coordinates $(x, g(f(x)))$ that fit on the grid. Draw a smooth curve through the points for the graph of $y = g(f(x))$.
There are no restrictions on x because both $f(x)$ and $g(x)$ are polynomial functions, which have domain $x \in \mathbb{R}$.

11. Given the functions $f(x) = \sqrt{3 + x}$, $g(x) = 1 - 2x^2$, and $k(x) = \frac{1}{x - 3}$, write an explicit equation for each combination.

a) $h(x) = f(g(x)) - k(x)$

$f(g(x)) = \sqrt{3 + 1 - 2x^2}$
$f(g(x)) = \sqrt{4 - 2x^2}$, $-\sqrt{2} \leq x \leq \sqrt{2}$
So, $h(x) = \sqrt{4 - 2x^2} - \frac{1}{x - 3}$, $-\sqrt{2} \leq x \leq \sqrt{2}$

b) $h(x) = g(f(x)) - f(g(x))$

$g(f(x)) = 1 - 2(\sqrt{3 + x})^2$
$g(f(x)) = 1 - 2(3 + x)$, $x \geq -3$
$g(f(x)) = -5 - 2x$, $x \geq -3$
So, $h(x) = -5 - 2x - \sqrt{4 - 2x^2}$, $-\sqrt{2} \leq x \leq \sqrt{2}$

c) $h(x) = k(g(x)) + k(f(x))$

$k(g(x)) = \dfrac{1}{1 - 2x^2 - 3}$

$k(g(x)) = \dfrac{1}{-2x^2 - 2}$

$k(f(x)) = \dfrac{1}{\sqrt{3 + x} - 3}$

So, $h(x) = \dfrac{1}{-2x^2 - 2} + \dfrac{1}{\sqrt{3 + x} - 3}$, $x \geq -3, x \neq 6$

d) $h(x) = g(g(x)) \cdot k(x)$

$g(g(x)) = 1 - 2(1 - 2x^2)^2$

$g(g(x)) = 1 - 2(1 - 4x^2 + 4x^4)$

$g(g(x)) = -1 + 8x^2 - 8x^4$

So, $h(x) = (-1 + 8x^2 - 8x^4)\left(\dfrac{1}{x - 3}\right), x \neq 3$

12. Given the functions $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3x - 18$, determine an explicit equation for each composite function then state its domain.

a) $f(g(x))$

In $f(x) = \sqrt{x}$, replace x with $x^2 - 3x - 18$.

$f(g(x)) = \sqrt{x^2 - 3x - 18}$

The domain of $g(x) = x^2 - 3x - 18$ is: $x \in \mathbb{R}$

The domain of $f(x) = \sqrt{x}$ is: $x \geq 0$.

So, $g(x) \geq 0$

And $x^2 - 3x - 18 \geq 0$

Solve the corresponding quadratic equation.

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x = 6$ or $x = -3$

Choose a value of $x < -3$, such as $x = -4$.

Use mental math to substitute $x = -4$ in $x^2 - 3x - 18 \geq 0$:

L.S $= 10$ R.S. $= 0$

So, the interval $x \leq -3$ is the correct interval. And the interval $x \geq 6$ will also be correct.

So, the domain of $f(g(x))$ is $x \leq -3$ or $x \geq 6$.

b) $g(f(x))$

In $g(x) = x^2 - 3x - 18$, replace x with $\sqrt{x}$.

$g(f(x)) = (\sqrt{x})^2 - 3\sqrt{x} - 18$

$g(f(x)) = x - 3\sqrt{x} - 18$

The domain of $f(x)$ is: $x \geq 0$

The domain of $g(x)$ is: $x \in \mathbb{R}$

So, the domain of $g(f(x))$ is: $x \geq 0$

13. For each function below

i) Determine possible functions f and g so that $y = f(g(x))$.

ii) Determine possible functions f, g, and h so that $y = f(g(h(x)))$.

a) $y = -3\sqrt{x + 4}$

Sample response:

i) $g(x) = x + 4$ and $f(x) = -3\sqrt{x}$

ii) $h(x) = x + 4$, $g(x) = \sqrt{x}$, and $f(x) = -3x$

b) $y = (2 - 5x)^3$

Sample response:

i) $g(x) = 2 - 5x$ and $f(x) = x^3$

ii) $h(x) = 5x$, $g(x) = 2 - x$, and $f(x) = x^3$